Information Bottleneck Learning Using Privileged Information for Visual Recognition (Supplementary Material)

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1. Introduction

In this report, we explain the lower bound and upper bound of our method [3]. We also add more experiments to support our model. You may refer to [2] for more information.

2. LMBPI bounds

2.1. Lower-bound LMIBPI

In addition to the proposed LMIBPI [3], we also deploy two more approaches, representing the upper and lower bounds of LMIBPI. The lower bound corresponds to eliminating the use of auxiliary information from LMIBPI. Computationally, this can be achieved very easily by setting $\gamma = 0$ in Eq. (8) of the main paper. For each of the experiments we have added a column indicated with `LB-LMIBPI`, which stands for lower-bound LMIBPI, and which represents this case.

2.2. Upper-bound LMIBPI

The upper-bound model for LMIBPI corresponds to the case for when main and auxiliary data are available at both training and testing time. We model this situation as in Figure 1. In particular, we allow the auxiliary data X^* to be compressed, and obtain T^*, as indicated by G_{in} in Figure 1. The desired output is identified by G_{out}, where T^* d-separates X^* and Y, and T d-separates X and Y. Also, we have that (T, T^*) d-separates (X, X^*) from Y. This means that we would like to have at the same time $I(X; Y|T) = 0$, $I(X^*; Y|T^*) = 0$, and $I(X, X^*; Y|T, T^*) = 0$. So, we should compress X and X^* as much as possible, provided that T and T^* retain all the information about Y.

The multi-information of G_{in} and G_{out} of Figure 1 is given by

$$\mathcal{I}^{G_{in}} = I(T; X) + I(T^*; X^*) + I(Y; X, X^*) , \quad (1)$$
$$\mathcal{I}^{G_{out}} = I(T; X) + I(T^*; X^*) + I(T, T^*; Y) . \quad (2)$$

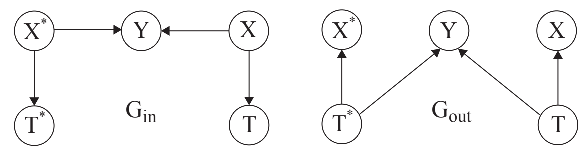

Figure 1. **Upper-bound IBPI.** Structural representation of G_{in} and G_{out} used by the information bottleneck method when main and auxiliary information are fused to provide the upper-bound of the IBPI method.

Since $I(Y; X, X^*)$ is constant, the functional for learning the optimal representation for T and T^* becomes

$$\mathcal{L}[q(T, T^*|X, X^*)] = I(T; X) + I(T^*, X^*) - \gamma I(T, T^*; Y) . \quad (3)$$

From (3) we derive the large-margin formulation of the upper-bound of LMIBPI. In particular, we restrict the search space for $q(T, T^*|X, X^*)$ by assuming that

$$T = \phi(X; A) , \quad (4)$$
$$T^* = \phi(X^*; A^*) , \quad (5)$$

where A and A^* are two suitable sets of parameters. Moreover, from T and T^* we aim at predicting the relevant information Y through the decision function given by

$$Y = \text{sign}\left(\left\langle w, \begin{bmatrix} T \\ T^* \end{bmatrix} \right\rangle + b\right) . \quad (6)$$

Therefore, the large-margin problem that we solve is the following

$$\begin{aligned} \min_{A,A^*,w,b,\xi_i} \; & I(T; X) + I(T^*; X^*) + \frac{\beta}{2}\|w\|^2 + \frac{C}{N}\sum_{i=1}^{N}\xi_i \\ \text{s.t.} \; & y_i\left(\left\langle w, \begin{bmatrix} \phi(x_i, A) \\ \phi(x_i^*, A^*) \end{bmatrix} \right\rangle + b\right) \geq 1 - \xi_i , \quad (7) \\ & \xi_i \geq 0 , \quad \forall i \in \{1, \cdots, N\} . \end{aligned}$$

The optimization of problem (7), which we refer to as *upper-bound LMIBPI (UP-LMIBPI)* can be carried out with techniques similar to those in the paper. In particular, we assume the relationships $T = AX$, and $T^* = A^*X^*$, where A and A^* are stochastic matrices with normalized columns. Although now we have to estimate A and A^*, the parameter γ of Eq. (8) in the paper has disappeared. The implementation of the optimization entails alternating between solving an SVM problem when A and A^* are known, and then keep every parameter fixed and update A, and subsequently update A^*. The update of A and A^* requires a set of equations similar to those in the main paper.

In the rest of this supplementary material we refer to the implementation of UP-LMIBPI as `UP-LMIBPI`, and for each experiment we report results corresponding to this case, where auxiliary information is available at testing time too (which actually makes it no-longer privileged!).

3. HMDB, ImageNet, and CGD2011 datasets

HMDB dataset. Table 1 provides the classification accuracy for the same experiment reported in Table 1 of the main paper, with the exception that the linear kernel here is replaced by the HIK kernel. Figure 3 shows the differences between the accuracies of `LMIBPI` versus `RankTr`, `SVM+`, `SVM2k-LUPI` and `KCCA-LUPI`. These plots highlight that `LMIBPI` compares favorably. Figure 4 shows how `LMIBPI` compares against `LB-LMIBPI` and `UB-LMIBPI`. If we consider the *performance gap* between lower and upper bound accuracies, and we identify with 0% the lower bound, and with 100% the upper bound, on average, using the auxiliary information allows recovering 47.1% of the performance gap in the linear kernel case, and 35.4% when HIK is used. Also, using HIK improves the average performance from $81.11 \pm 5.80\%$ to $83.71 \pm 5.75\%$.

ImageNet dataset. Table 2 provides the classification accuracy for the same experiment reported in Table 2 of the main paper, with the exception that the linear kernel here is replaced by the HIK kernel. Figure 5 shows the differences between the accuracies of `LMIBPI` versus `RankTr`, `SVM+`, `SVM2k-LUPI` and `KCCA-LUPI`. These plots highlight that `LMIBPI` compares favorably. Figure 6 shows how `LMIBPI` compares against `LB-LMIBPI` and `UB-LMIBPI`. On average, using the auxiliary information allows recovering 47.4% of the performance gap in the linear kernel case, and 50.6% when HIK is used. Also, using HIK improves the average performance from $66.94 \pm 6.39\%$ to $69.92 \pm 6.39\%$.

CGD2011 dataset. For auxiliary information we used the positions of the joints, which is part of the CGD2011 dataset [1]. In particular, for a video sequence with a gesture, we extract a histogram of the joint positions, accumulated over all the frames of the sequence. For instance, as shown in Figure 2, at every frame we place a spatial grid aligned with the head position of an individual, and bin the position of each of the joints with respect to the grid. This is repeated for every frame, and the resulting count accumulated over the entire video is normalized, and produces a histogram with 100 bins.

Table 3 provides the classification accuracy for the same experiment reported in Table 3 of the main paper, with the exception that the linear kernel here is replaced by the HIK kernel. Figure 7 shows the differences between the accuracies of `LMIBPI` versus `RankTr`, `SVM+`, `SVM2k-LUPI` and `KCCA-LUPI`. These plots highlight that `LMIBPI` compares favorably. Figure 8 shows how `LMIBPI` compares against `LB-LMIBPI` and `UB-LMIBPI`. On average, using the auxiliary information allows recovering 41.6% of the performance gap in the linear kernel case, and 50.7% when HIK is used. Also, using HIK improves the average performance from $61.42 \pm 4.60\%$ to $65.50 \pm 3.55\%$.

Figure 9 gives a comprehensive outlook of how `LMIBPI` compares with the other single-view and LUPI classifiers. In general, `LMIBPI` outperforms the competition. For all the three datasets, HMDB, ImageNet, and CGD2011, using a non-linear kernel has led to a performance improvement, and for ImageNet and CGD2011 this has also led to a further performance improvement of `LMIBPI` with respect to the other approaches.

Finally, for scientific honesty, we note that in 2 cases for HMDB with HIK kernel, 2 cases for ImageNet with linear kernel, and 1 case for CGD2011 with linear kernel, on average, `LMIBPI` performs slightly below `LB-LMIBPI`. These are a few unfortunate cases where the optimization on average converges to a local minimum that leads to a worse solution than the corresponding single-view classifier.

4. AwA dataset

Table 4 completes Table 4 from the main paper in that it adds the performance of `SVM`, `RankTr` [4], `SVM+`, and `LIR` [5]. Those were omitted from the main paper because of lack of space. Finally, Table 5 repeats the same experiment of Table 4 where the linear kernel has been replaced with a Gaussian kernel. The average AP is 88.38 for the linear kernel and also for the Gaussian kernel. Therefore, switching to a non-linear kernel has not improved the performance. This is likely due to the use of SURF features, which have a very high dimension, and are known to work well with linear kernels.

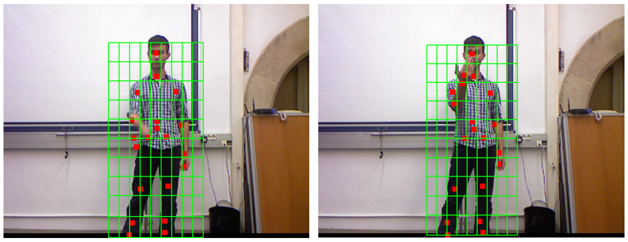

Figure 2. **CGD2011 dataset.** Samples from the CGD2011 dataset with joint information superimposed (red squares), together with a (green) grid visualizing the binning of the joint positions.

References

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	LB-LMIBPI	SVM	SVM-R	RankTr	SVM+	SVM2k-LUPI	KCCA-LUPI	LMIBPI	UB-LMIBPI	SVM2k	KCCA
brush hair	91.67 ± 3.42	90.66 ± 3.06	73.33 ± 4.15	82.33 ± 4.09	**93.66 ± 1.72**	85.33 ± 5.56	90.33 ± 3.66	93.50 ± 2.42	93.83 ± 1.77	88.33 ± 9.52	91.33 ± 3.99
dive	88.50 ± 4.19	81.16 ± 4.44	83.83 ± 4.84	82.50 ± 3.16	84.50 ± 3.68	79.00 ± 5.73	83.00 ± 4.14	**86.67 ± 2.61**	90.83 ± 3.26	83.50 ± 3.96	83.50 ± 2.65
drink	76.83 ± 4.26	76.66 ± 6.52	68.83 ± 8.99	71.16 ± 8.01	75.16 ± 7.13	70.50 ± 3.33	77.16 ± 7.37	**78.83 ± 5.21**	81.00 ± 4.92	77.16 ± 5.44	79.16 ± 6.09
eat	75.00 ± 5.27	73.00 ± 5.70	72.83 ± 5.33	73.50 ± 5.05	75.50 ± 5.82	70.50 ± 4.23	74.33 ± 5.94	**78.00 ± 4.07**	79.33 ± 3.16	76.83 ± 8.02	77.33 ± 5.56
golf	84.50 ± 2.36	82.33 ± 3.61	78.83 ± 4.84	85.33 ± 5.43	86.16 ± 4.16	77.16 ± 6.18	84.33 ± 3.70	**88.17 ± 4.87**	89.83 ± 5.00	88.00 ± 3.02	89.00 ± 3.35
hug	86.00 ± 4.79	83.83 ± 4.84	74.83 ± 4.87	**87.33 ± 3.70**	86.33 ± 3.49	80.33 ± 6.17	83.83 ± 4.01	86.83 ± 4.34	87.50 ± 5.05	85.50 ± 4.23	86.16 ± 3.77
jump	80.50 ± 4.85	77.33 ± 7.78	78.33 ± 4.08	75.50 ± 4.30	79.66 ± 4.21	69.50 ± 5.62	**80.83 ± 6.24**	80.67 ± 4.98	84.33 ± 7.17	77.50 ± 4.46	81.50 ± 6.95
pick	73.17 ± 5.47	70.16 ± 11.1	65.83 ± 5.22	68.33 ± 3.33	72.33 ± 4.45	60.16 ± 7.09	70.66 ± 5.10	**74.83 ± 5.00**	75.17 ± 7.47	72.83 ± 4.44	73.50 ± 5.95
punch	88.10 ± 2.95	84.00 ± 5.67	83.83 ± 4.90	84.50 ± 4.44	**88.16 ± 5.58**	79.33 ± 4.66	86.66 ± 4.90	87.67 ± 4.10	88.83 ± 4.38	82.00 ± 5.76	82.33 ± 5.67
sit	78.00 ± 4.76	74.16 ± 6.15	76.50 ± 6.82	75.33 ± 4.14	76.33 ± 6.02	69.33 ± 7.70	76.33 ± 5.14	**82.00 ± 5.58**	83.17 ± 5.58	73.50 ± 6.82	75.33 ± 6.22

Table 1. **HMDB dataset - HIK.** Classification accuracies for one-vs-all binary classifications. The HOF features represented main data, and HOG features auxiliary data. Best accuracies are highlighted in boldface.

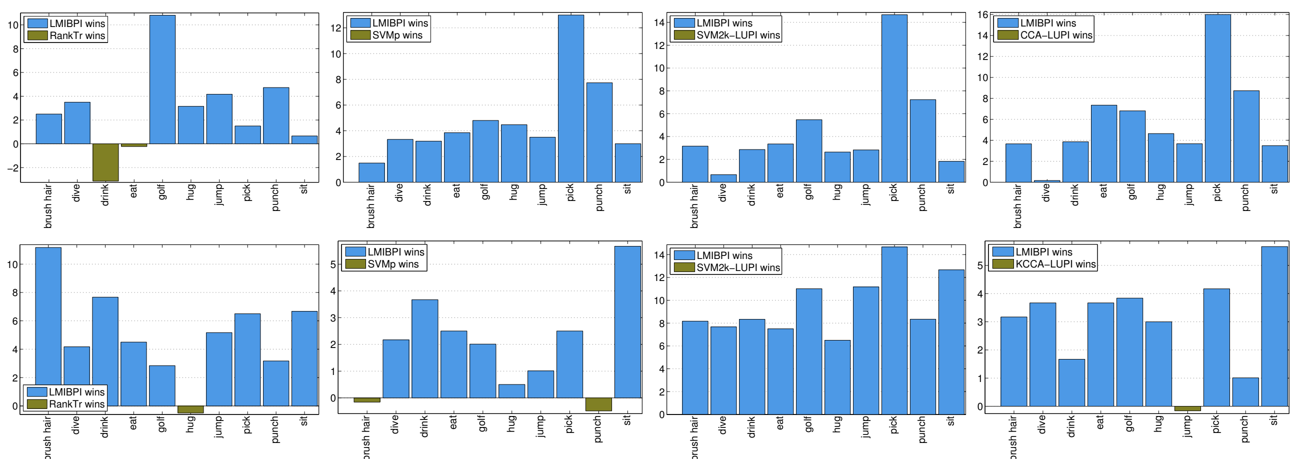

Figure 3. **HMDB dataset.** Each row shows the plots of the differences between the classification accuracy of LMIBPI versus RankTr, SVM+, SVM2k-LUPI, and CCA-LUPI, respectively. The top row refers to the use of the linear kernel. The bottom row refers to the use of the HIK kernel.

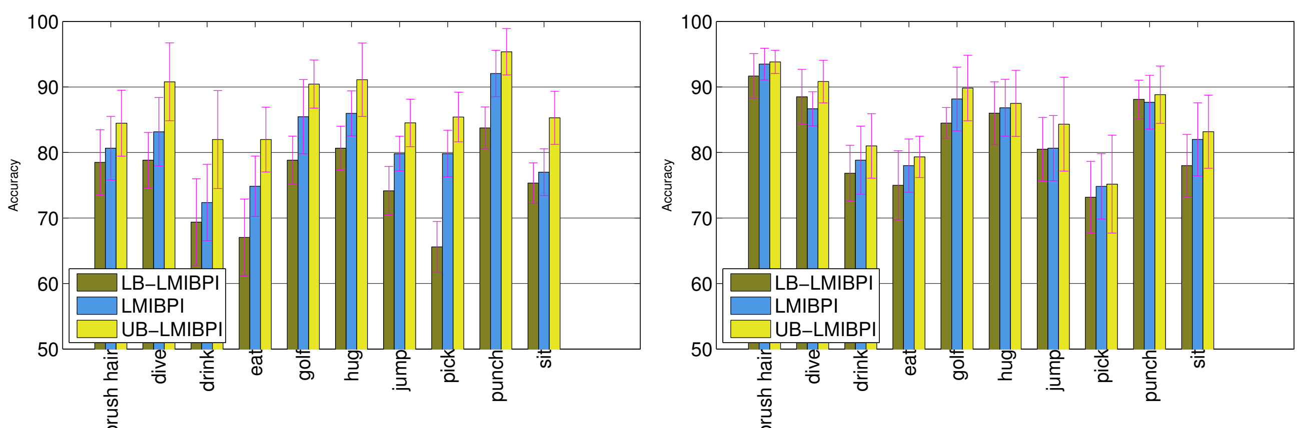

Figure 4. **HMDB dataset.** Comparison between the classification accuracy of LMIBPI versus the corresponding single-view classifier (lower bound) LB-LMIBPI, and two-view classifier (upper bound) UB-LMIBPI. The left plot refers to the use of the linear kernel. The right plot refers to the use of the HIK kernel.

	LB-LMIBPI	SVM	SVM-R	RankTr	SVM+	SVM2k-LUPI	KCCA-LUPI	LMIBPI	UB-LMIBPI	SVM2k	KCCA
Thunder snake	54.32 ± 2.21	58.21 ± 1.95	55.35 ± 3.45	**62.31 ± 3.21**	61.59 ± 1.63	56.74 ± 3.09	59.71 ± 2.15	60.52 ± 3.21	64.73 ± 3.21	59.43 ± 2.49	58.40 ± 2.75
Ringneck snake	65.23 ± 1.78	64.60 ± 2.57	56.84 ± 4.02	64.32 ± 2.15	65.49 ± 2.86	61.26 ± 2.67	61.93 ± 1.53	**67.51 ± 2.11**	68.32 ± 2.56	62.88 ± 2.96	62.80 ± 2.47
Hognose snake	58.43 ± 2.21	60.00 ± 1.38	59.14 ± 2.65	61.22 ± 1.83	59.52 ± 2.29	54.78 ± 2.72	57.47 ± 3.01	**63.72 ± 1.89**	66.51 ± 3.45	55.90 ± 2.70	59.25 ± 1.93
Green snake	69.25 ± 1.32	71.59 ± 1.65	60.66 ± 4.20	72.23 ± 2.68	70.34 ± 1.21	62.42 ± 7.44	65.03 ± 1.43	**73.21 ± 2.63**	75.42 ± 2.87	66.97 ± 7.13	70.46 ± 1.80
King snake	64.41 ± 3.22	63.46 ± 1.48	53.00 ± 2.74	66.21 ± 3.25	65.76 ± 1.61	55.05 ± 4.50	60.74 ± 1.32	**67.31 ± 3.25**	68.22 ± 5.36	56.86 ± 5.81	61.72 ± 2.60
Garter snake	71.22 ± 2.33	69.27 ± 2.52	60.17 ± 4.26	**72.33 ± 3.36**	70.34 ± 2.37	65.29 ± 3.59	64.40 ± 1.86	71.44 ± 4.15	73.25 ± 4.32	68.88 ± 2.13	68.49 ± 1.32
Water snake	71.31 ± 3.25	72.32 ± 1.84	62.75 ± 2.98	73.45 ± 4.51	71.88 ± 1.23	69.65 ± 1.85	65.98 ± 1.64	**74.41 ± 2.63**	76.85 ± 3.25	69.76 ± 1.74	70.05 ± 1.68
Vine snake	79.05 ± 2.51	79.12 ± 2.27	67.64 ± 5.86	78.32 ± 3.72	79.05 ± 1.96	76.21 ± 4.15	73.67 ± 1.25	**80.11 ± 3.65**	83.22 ± 2.31	77.55 ± 2.97	78.13 ± 1.74
Night snake	56.33 ± 3.11	55.69 ± 3.22	52.90 ± 2.10	**59.23 ± 2.23**	57.27 ± 1.74	54.60 ± 2.96	55.91 ± 3.08	58.32 ± 4.11	60.11 ± 4.28	55.65 ± 2.93	55.43 ± 2.57
Boa constrictor	64.80 ± 2.11	64.69 ± 1.78	52.61 ± 2.84	63.21 ± 1.80	65.26 ± 1.81	57.18 ± 5.62	63.00 ± 1.07	**66.21 ± 2.56**	68.42 ± 3.35	60.04 ± 6.85	64.52 ± 2.45
Rock python	62.22 ± 1.78	61.99 ± 1.63	52.86 ± 2.05	61.55 ± 2.35	60.57 ± 1.75	54.07 ± 4.34	58.53 ± 1.76	**64.32 ± 3.89**	68.23 ± 2.36	55.87 ± 4.91	60.59 ± 2.38
Indian cobra	66.51 ± 1.96	67.23 ± 2.33	59.73 ± 5.50	67.23 ± 4.36	**68.57 ± 1.95**	62.42 ± 4.82	65.19 ± 2.14	67.89 ± 2.75	71.08 ± 4.83	64.23 ± 3.52	65.98 ± 2.73
Green mamba	69.22 ± 1.32	68.18 ± 1.97	60.58 ± 4.23	71.22 ± 1.25	69.59 ± 1.68	64.79 ± 5.25	64.79 ± 2.03	**74.32 ± 1.56**	78.31 ± 3.96	65.82 ± 4.55	67.50 ± 1.86
Sea snake	78.53 ± 3.21	77.90 ± 9.05	69.07 ± 7.32	79.22 ± 3.38	81.48 ± 1.44	63.47 ± 9.81	75.70 ± 1.08	**82.32 ± 3.18**	83.41 ± 3.57	65.59 ± 9.21	79.07 ± 1.03
Horned viper	71.32 ± 4.51	70.64 ± 1.65	59.23 ± 4.16	71.23 ± 2.68	70.84 ± 1.29	60.01 ± 6.64	66.00 ± 2.21	**73.45 ± 3.75**	76.77 ± 2.58	64.05 ± 7.02	69.29 ± 1.79
Diamondback	68.31 ± 2.11	67.45 ± 2.09	58.00 ± 5.02	69.35 ± 3.65	68.55 ± 2.54	58.48 ± 5.65	63.44 ± 2.56	**71.22 ± 2.86**	72.31 ± 1.36	62.14 ± 4.87	65.32 ± 2.20
Sidewinder	70.21 ± 1.17	69.62 ± 2.10	63.23 ± 6.20	69.58 ± 4.11	71.39 ± 2.62	66.79 ± 3.49	67.54 ± 2.70	**72.36 ± 1.98**	78.33 ± 2.35	68.18 ± 2.34	69.10 ± 2.83

Table 2. **ImageNet dataset - HIK.** Classification accuracies for one-vs-all binary classifications. The BoW from the whole image represented main data, and the BoW from the bounding box region auxiliary data. Best accuracies are highlighted in boldface.

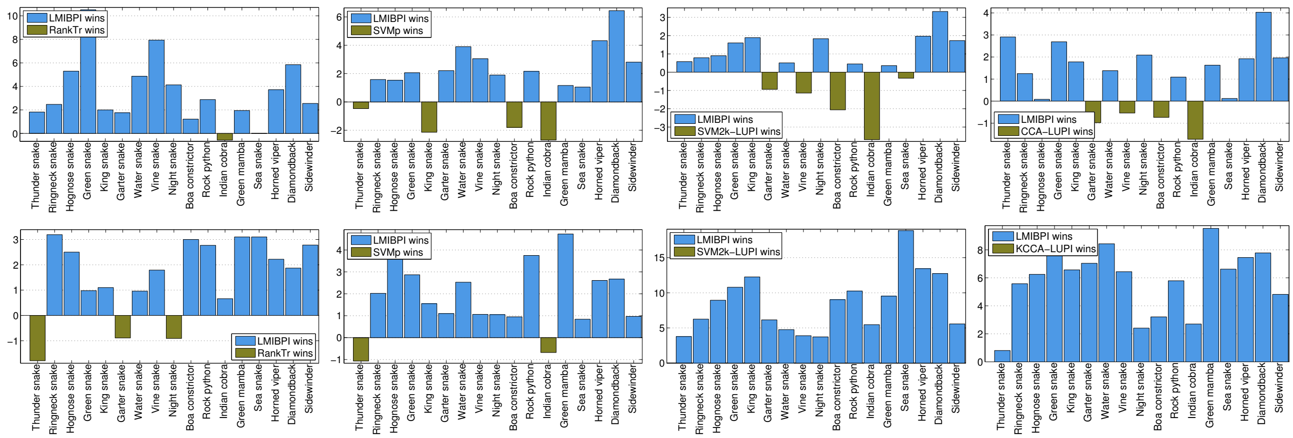

Figure 5. **ImageNet dataset.** Each row shows the plots of the differences between the classification accuracy of `LMIBPI` versus `RankTr`, `SVM+`, `SVM2k-LUPI`, and `CCA-LUPI`, respectively. The top row refers to the use of the linear kernel. The bottom row refers to the use of the HIK kernel.

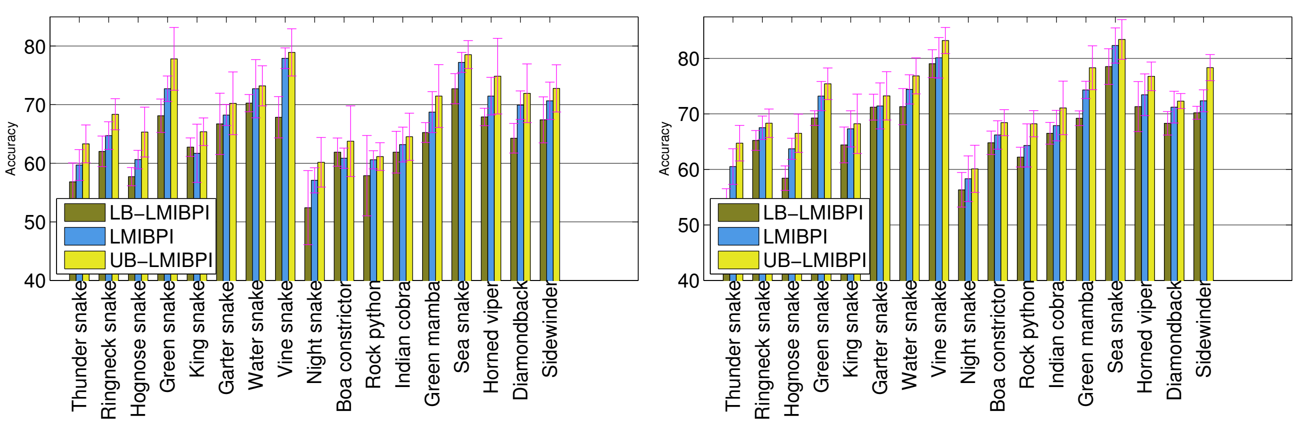

Figure 6. **ImageNet dataset.** Comparison between the classification accuracy of `LMIBPI` versus the corresponding single-view classifier (lower bound) `LB-LMIBPI`, and two-view classifier (upper bound) `UB-LMIBPI`. The left plot refers to the use of the linear kernel. The right plot refers to the use of the HIK kernel.

	LB-LMIBPI	SVM	SVM-R	RankTr	SVM+	SVM2k-LUPI	KCCA-LUPI	LMIBPI	UB-LMIBPI	SVM2k	KCCA
vieniqui	55.78 ± 4.72	50.22 ± 2.88	53.94 ± 3.70	52.61 ± 3.91	52.22 ± 4.47	51.38 ± 5.89	54.38 ± 3.88	**60.27 ± 3.77**	62.27 ± 5.23	56.16 ± 3.97	55.44 ± 3.90
prendere	59.44 ± 2.53	56.05 ± 2.82	50.77 ± 1.36	54.39 ± 2.95	58.61 ± 2.91	48.27 ± 3.39	58.50 ± 2.23	**61.27 ± 1.83**	64.61 ± 3.25	58.05 ± 6.15	61.38 ± 4.21
sonostufo	60.39 ± 3.27	56.11 ± 2.60	55.27 ± 4.64	56.72 ± 4.21	59.88 ± 4.33	53.83 ± 6.30	58.66 ± 3.95	**62.94 ± 2.79**	63.73 ± 3.50	67.55 ± 5.16	63.00 ± 4.31
chevuoi	60.17 ± 3.69	58.27 ± 2.88	59.22 ± 3.37	58.11 ± 2.38	58.66 ± 3.03	56.66 ± 3.20	59.33 ± 3.05	**65.05 ± 3.56**	67.86 ± 3.65	66.66 ± 3.12	67.22 ± 4.32
daccordo	66.06 ± 4.42	67.05 ± 3.53	63.88 ± 5.33	61.83 ± 3.07	67.94 ± 2.52	66.05 ± 8.54	66.70 ± 2.18	**71.66 ± 2.84**	73.44 ± 5.72	77.50 ± 5.17	78.50 ± 2.45
perfetto	66.94 ± 2.42	66.55 ± 4.51	62.33 ± 4.16	62.05 ± 3.52	66.33 ± 2.74	63.33 ± 8.61	63.00 ± 3.32	**68.44 ± 2.85**	71.11 ± 2.82	68.16 ± 5.39	60.55 ± 3.16
vattene	67.61 ± 1.56	64.50 ± 2.80	61.72 ± 3.04	60.28 ± 2.67	64.61 ± 2.83	62.11 ± 3.27	63.77 ± 2.92	**69.05 ± 2.63**	72.33 ± 3.45	73.44 ± 3.87	67.88 ± 3.77
basta	67.06 ± 5.14	67.84 ± 4.47	63.66 ± 6.52	63.44 ± 6.11	68.05 ± 2.76	59.66 ± 6.03	69.05 ± 3.46	**69.33 ± 3.01**	73.33 ± 2.80	71.11 ± 5.73	79.94 ± 4.07
buonissimo	57.78 ± 4.23	55.05 ± 3.84	57.33 ± 3.10	54.55 ± 3.83	56.61 ± 4.33	54.66 ± 2.78	56.66 ± 3.71	**63.50 ± 3.77**	65.96 ± 3.63	65.38 ± 6.00	60.66 ± 5.35
cheduepalle	67.61 ± 2.85	66.72 ± 2.44	65.88 ± 2.11	61.94 ± 5.63	66.88 ± 3.64	62.00 ± 4.99	63.50 ± 4.00	**70.33 ± 2.40**	74.44 ± 1.88	75.66 ± 2.61	70.55 ± 3.96
cosatifarei	65.88 ± 4.12	63.55 ± 3.20	59.72 ± 4.12	59.00 ± 3.85	61.77 ± 3.30	58.11 ± 7.29	61.38 ± 4.38	**68.33 ± 3.37**	73.05 ± 3.36	67.11 ± 6.24	62.44 ± 4.80
fame	62.05 ± 5.25	56.68 ± 2.31	57.88 ± 2.97	58.83 ± 3.70	63.22 ± 4.87	52.94 ± 5.09	57.05 ± 2.62	**64.94 ± 1.86**	67.00 ± 5.71	66.22 ± 4.37	59.50 ± 4.22
noncenepiu	58.50 ± 4.32	53.22 ± 3.30	53.16 ± 1.83	55.61 ± 3.44	54.88 ± 4.22	53.50 ± 5.11	55.44 ± 3.74	**61.66 ± 3.26**	63.72 ± 3.88	60.83 ± 4.35	53.94 ± 2.36
furbo	63.44 ± 3.12	67.66 ± 3.27	63.33 ± 5.47	61.66 ± 3.29	65.88 ± 3.63	62.88 ± 4.97	63.55 ± 2.91	**68.55 ± 1.61**	72.22 ± 3.69	67.83 ± 3.35	60.88 ± 6.55
combinato	63.83 ± 2.12	59.44 ± 2.91	56.99 ± 2.54	59.78 ± 1.68	64.16 ± 1.92	59.83 ± 4.54	59.72 ± 1.98	**65.72 ± 4.19**	69.34 ± 2.32	78.16 ± 6.64	76.50 ± 1.65
freganiente	59.13 ± 3.25	51.94 ± 2.52	56.11 ± 3.22	54.89 ± 4.82	54.88 ± 3.52	52.05 ± 5.07	52.33 ± 2.34	**62.61 ± 2.61**	67.88 ± 3.23	59.16 ± 2.97	57.83 ± 2.54
seipazzo	60.56 ± 4.12	56.55 ± 4.11	56.77 ± 4.30	53.16 ± 6.42	55.05 ± 2.81	50.50 ± 5.74	57.44 ± 2.50	**63.55 ± 4.08**	65.96 ± 4.53	69.00 ± 5.28	64.88 ± 4.28
tantotempo	66.88 ± 4.15	65.72 ± 2.59	56.56 ± 3.61	59.11 ± 2.58	68.05 ± 1.94	63.55 ± 6.33	62.00 ± 2.99	**68.11 ± 2.38**	72.22 ± 4.78	75.38 ± 3.84	72.33 ± 4.03
messidaccordo	61.38 ± 3.80	60.66 ± 4.74	55.33 ± 2.64	58.33 ± 2.61	63.22 ± 4.10	58.61 ± 4.90	60.88 ± 4.67	**65.05 ± 2.72**	67.11 ± 3.21	59.27 ± 4.85	56.88 ± 6.03
ok	56.33 ± 1.22	53.15 ± 2.39	53.72 ± 2.49	52.55 ± 3.93	54.61 ± 3.56	50.66 ± 5.85	55.27 ± 3.18	**59.61 ± 3.43**	62.44 ± 2.65	56.27 ± 6.26	63.55 ± 2.64

Table 3. **CGD2011 dataset - HIK.** Classification accuracies for one-vs-all binary classifications. The HOF features represented main data, and histograms of joint positions were used as auxiliary data. Best accuracies are highlighted in boldface.

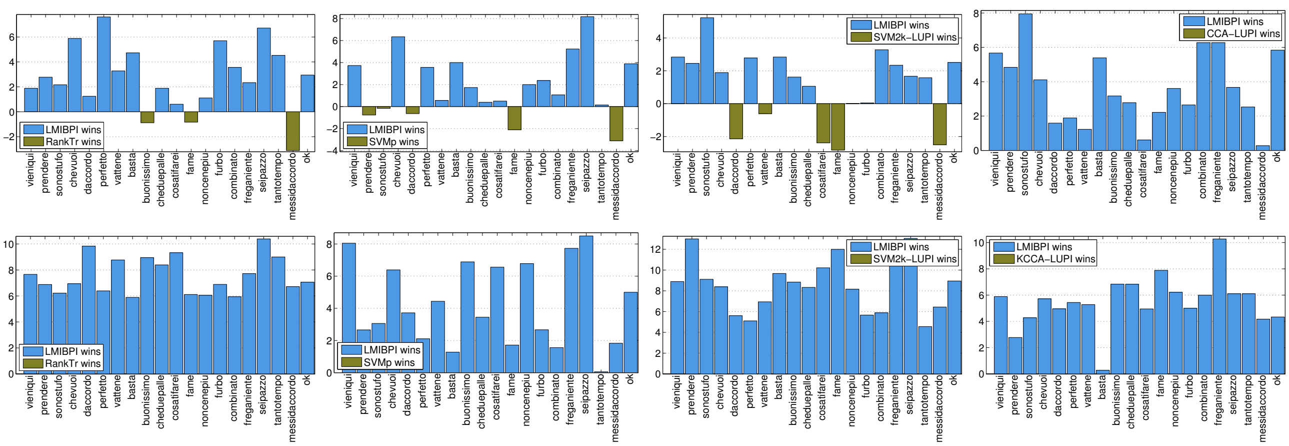

Figure 7. **CGD2011 dataset.** Each row shows the plots of the differences between the classification accuracy of LMIBPI versus RankTr, SVM+, SVM2k-LUPI, and CCA-LUPI, respectively. The top row refers to the use of the linear kernel. The bottom row refers to the use of the HIK kernel.

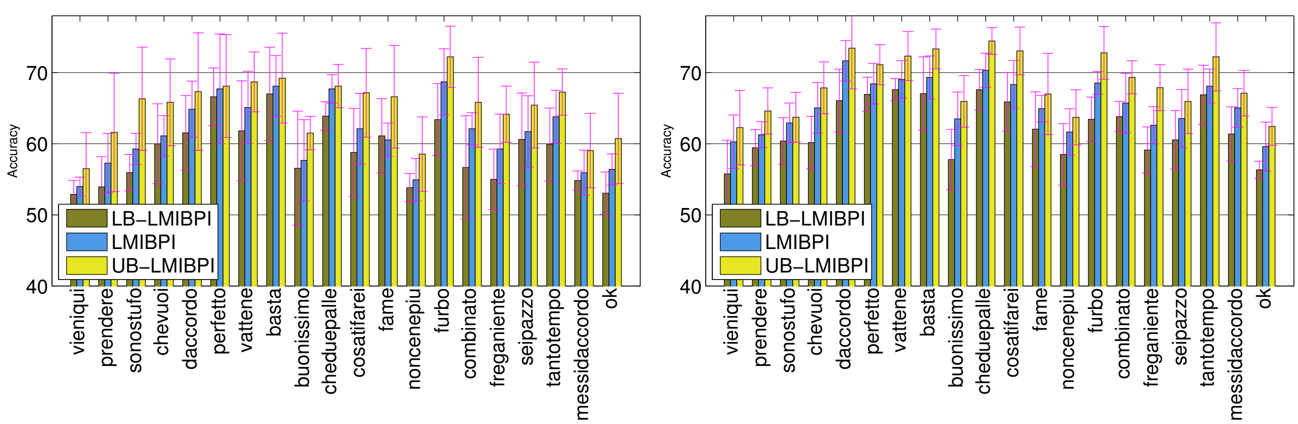

Figure 8. **CGD2011 dataset.** Comparison between the classification accuracy of LMIBPI versus the corresponding single-view classifier (lower bound) LB-LMIBPI, and two-view classifier (upper bound) UB-LMIBPI. The left plot refers to the use of the linear kernel. The right plot refers to the use of the HIK kernel.

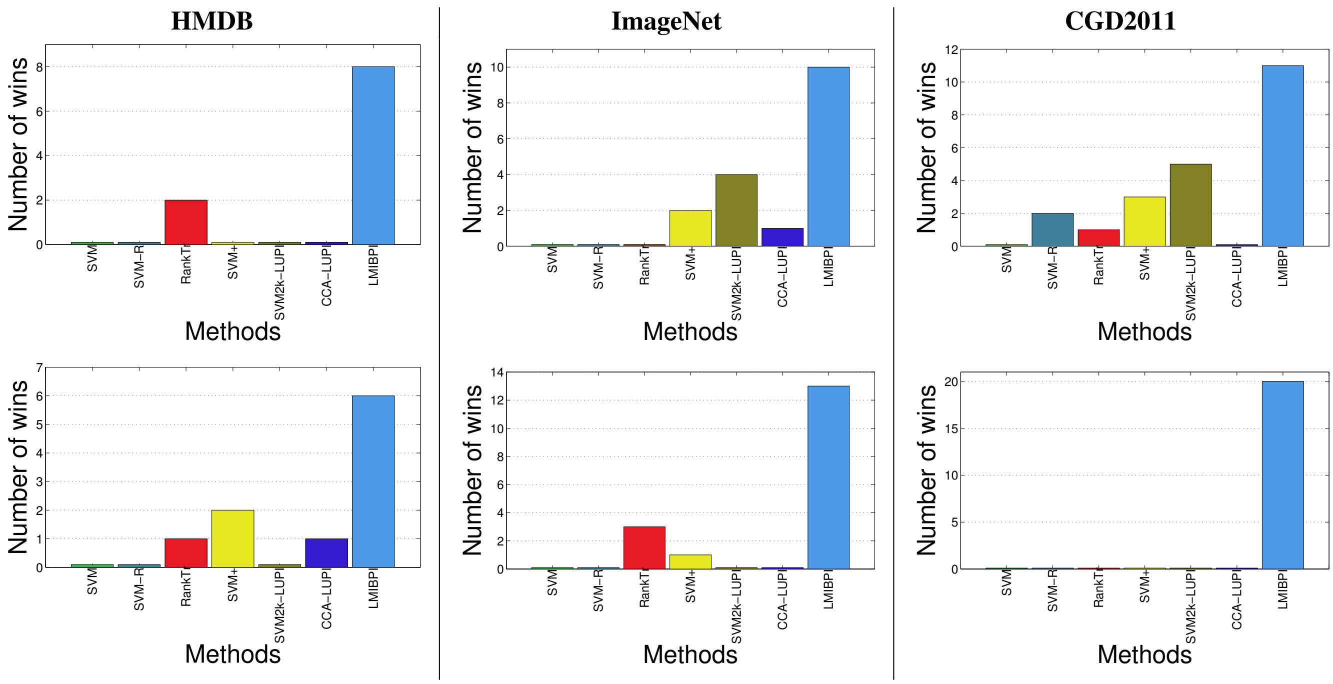

Figure 9. **HMDB, ImageNet and CGD2011 datasets.** Plots representing the number of wins accumulated by the single-view and the LUPI classifiers, namely `SVM`, `SVM-R`, and `RankTr`, `SVM+`, `SVM2k-LUPI`, `KCCA-LUPI`, `LMIBPI`. The top row refers to the use of the linear kernel. The bottom row refers to the use of the HIK kernel. The first column refers to the HMDB dataset. The middle column refers to the ImageNet dataset. The right column refers tot he CGD2011 dataset.

		SVM	RankTr	SVM+	LIR	LMIBPI
1	*Chimpanzee versus Giant panda*	88.88 ± 0.51	**89.33 ± 0.50**	88.07 ± 0.57	88.28 ± 0.47	88.32 ± 0.33
2	*Chimpanzee versus Leopard*	93.74 ± 0.26	93.70 ± 0.23	93.49 ± 0.29	93.36 ± 0.15	**94.05 ± 0.10**
3	*Chimpanzee versus Persian cat*	90.14 ± 0.40	91.00 ± 0.39	89.88 ± 0.42	**91.59 ± 0.40**	90.76 ± 0.19
4	*Chimpanzee versus Pig*	85.64 ± 0.57	86.08 ± 0.43	85.19 ± 0.53	83.74 ± 0.35	**87.32 ± 0.17**
5	*Chimpanzee versus Hippopotamus*	86.40 ± 0.55	86.92 ± 0.45	86.31 ± 0.59	89.63 ± 0.31	**90.21 ± 0.12**
6	*Chimpanzee versus Humpback whale*	98.03 ± 0.18	98.08 ± 0.18	97.74 ± 0.22	**98.30 ± 0.16**	97.76 ± 0.26
7	*Chimpanzee versus Raccoon*	87.01 ± 0.46	87.07 ± 0.48	86.64 ± 0.47	85.90 ± 0.63	**88.21 ± 0.27**
8	*Chimpanzee versus Rat*	85.42 ± 0.53	**86.67 ± 0.56**	84.83 ± 0.68	85.43 ± 0.48	85.31 ± 0.29
9	*Chimpanzee versus Seal*	91.74 ± 0.39	91.54 ± 0.43	91.10 ± 0.59	92.78 ± 0.42	**93.11 ± 0.23**
10	*Giant panda versus Leopard*	93.71 ± 0.38	93.76 ± 0.29	**94.03 ± 0.28**	92.81 ± 0.48	92.95 ± 0.20
11	*Giant panda versus Persian cat*	92.55 ± 0.41	92.57 ± 0.43	92.66 ± 0.32	**93.75 ± 0.29**	92.82 ± 0.32
12	*Giant panda versus Pig*	86.64 ± 0.45	86.22 ± 0.52	86.55 ± 0.40	84.19 ± 0.69	**86.71 ± 0.40**
13	*Giant panda versus Hippopotamus*	90.04 ± 0.56	90.89 ± 0.36	89.93 ± 0.56	**91.27 ± 0.35**	91.12 ± 0.29
14	*Giant panda versus Humpback whale*	98.38 ± 0.17	98.53 ± 0.15	98.11 ± 0.19	98.67 ± 0.11	**98.82 ± 0.14**
15	*Giant panda versus Raccoon*	**89.36 ± 0.44**	88.66 ± 0.60	89.06 ± 0.49	86.90 ± 0.74	89.21 ± 0.30
16	*Giant panda versus Rat*	88.49 ± 0.49	87.53 ± 0.51	87.86 ± 0.48	88.76 ± 0.37	**89.13 ± 0.25**
17	*Giant panda versus Seal*	92.81 ± 0.32	92.40 ± 0.40	92.59 ± 0.38	93.32 ± 0.31	**93.81 ± 0.19**
18	*Leopard versus Persian cat*	95.08 ± 0.25	**95.26 ± 0.25**	94.93 ± 0.24	**95.26 ± 0.22**	94.97 ± 0.22
19	*Leopard versus Pig*	88.55 ± 0.28	**88.90 ± 0.28**	88.37 ± 0.36	85.34 ± 0.50	87.31 ± 0.21
20	*Leopard versus Hippopotamus*	**92.98 ± 0.29**	92.86 ± 0.26	92.73 ± 0.31	92.54 ± 0.28	92.71 ± 0.16
21	*Leopard versus Humpback whale*	98.49 ± 0.30	98.63 ± 0.23	98.27 ± 0.31	**98.83 ± 0.11**	98.61 ± 0.26
22	*Leopard versus Raccoon*	80.31 ± 0.75	79.84 ± 0.59	79.94 ± 0.73	**81.31 ± 0.67**	80.12 ± 0.22
23	*Leopard versus Rat*	88.74 ± 0.35	89.27 ± 0.28	88.92 ± 0.35	89.93 ± 0.28	**90.13 ± 0.21**
24	*Leopard versus Seal*	93.87 ± 0.36	94.30 ± 0.36	93.74 ± 0.37	94.12 ± 0.21	**95.18 ± 0.33**
25	*Persian cat versus Pig*	81.55 ± 0.59	81.68 ± 0.46	81.45 ± 0.57	**82.60 ± 0.58**	82.27 ± 0.24
26	*Persian cat versus Hippopotamus*	92.42 ± 0.34	**92.82 ± 0.30**	92.33 ± 0.33	92.00 ± 0.49	92.38 ± 0.32
27	*Persian cat versus Humpback whale*	95.92 ± 0.29	95.84 ± 0.30	95.45 ± 0.38	97.36 ± 0.15	**97.42 ± 0.25**
28	*Persian cat versus Raccoon*	90.19 ± 0.40	90.38 ± 0.39	90.31 ± 0.41	**91.72 ± 0.34**	91.24 ± 0.18
29	*Persian cat versus Rat*	67.19 ± 0.60	69.07 ± 0.48	67.56 ± 0.63	69.62 ± 0.84	**70.49 ± 0.45**
30	*Persian cat versus Seal*	84.79 ± 0.60	85.66 ± 0.49	84.46 ± 0.54	88.38 ± 0.44	**88.41 ± 0.36**
31	*Pig versus Hippopotamus*	74.42 ± 0.48	75.57 ± 0.58	73.47 ± 0.55	**77.75 ± 0.51**	73.42 ± 0.12
32	*Pig versus Humpback whale*	96.01 ± 0.33	95.93 ± 0.37	95.75 ± 0.30	**96.85 ± 0.18**	95.93 ± 0.12
33	*Pig versus Raccoon*	77.73 ± 0.80	79.13 ± 0.63	76.96 ± 0.85	81.61 ± 0.71	**82.19 ± 0.15**
34	*Pig versus Rat*	68.66 ± 0.76	70.77 ± 0.73	68.58 ± 0.41	72.47 ± 0.55	**73.31 ± 0.25**
35	*Pig versus Seal*	77.91 ± 0.71	79.26 ± 0.77	77.32 ± 0.73	82.61 ± 0.55	**83.11 ± 0.43**
36	*Hippopotamus versus Humpback whale*	**92.19 ± 0.44**	92.17 ± 0.44	91.64 ± 0.60	91.08 ± 0.63	90.11 ± 0.28
37	*Hippopotamus versus Raccoon*	85.54 ± 0.60	**85.84 ± 0.70**	85.03 ± 0.60	85.72 ± 0.63	84.46 ± 0.36
38	*Hippopotamus versus Rat*	84.49 ± 0.39	85.62 ± 0.48	84.25 ± 0.37	85.91 ± 0.48	**86.11 ± 0.26**
39	*Hippopotamus versus Seal*	69.79 ± 0.83	**70.83 ± 0.79**	69.43 ± 0.84	69.79 ± 0.70	70.49 ± 0.41
40	*Humpback whale versus Raccoon*	96.67 ± 0.28	96.90 ± 0.29	96.57 ± 0.31	**97.34 ± 0.20**	96.97 ± 0.27
41	*Humpback whale versus Rat*	94.52 ± 0.19	**94.56 ± 0.22**	93.97 ± 0.24	92.95 ± 0.68	93.89 ± 0.19
42	*Humpback whale versus Seal*	84.60 ± 0.49	84.81 ± 0.38	84.24 ± 0.49	85.91 ± 0.57	**86.13 ± 0.17**
43	*Raccoon versus Rat*	77.65 ± 0.64	78.61 ± 0.72	78.36 ± 0.54	**80.00 ± 0.57**	79.63 ± 0.14
44	*Raccoon versus Seal*	91.43 ± 0.36	91.51 ± 0.40	91.37 ± 0.38	89.21 ± 0.43	**91.63 ± 0.36**
45	*Rat versus Seal*	78.45 ± 0.65	**79.88 ± 0.69**	78.28 ± 0.75	79.02 ± 0.50	79.21 ± 0.28
	Average	87.28	87.92	87.53	88.13	**88.38**

Table 4. **AwA dataset.** AP results for one-vs-one classification. Best average precisions are highlighted in boldface. The red boldface numbers indicate when the improvement of the LMIBPI method over the second best value was significant according to the z-test. The table refers to the case where we use 50 and 200 samples per class for training and testing, respectively. The values for the columns indicated as SVM, RankTr, SVM+, and LIR have been reported directly from [5] and from the supplementary material of [4].

		SVM	SVM+	SVM2k-LUPI	KCCA-LUPI	LMIBPI
1	*Chimpanzee versus Giant panda*	**87.96** ± **0.18**	87.81 ± 0.08	87.89 ± 0.42	87.60 ± 0.20	86.48 ± 0.46
2	*Chimpanzee versus Leopard*	**95.15** ± **0.56**	92.12 ± 0.33	93.80 ± 0.29	92.39 ± 0.30	92.90 ± 0.26
3	*Chimpanzee versus Persian cat*	88.20 ± 0.13	92.30 ± 0.04	90.60 ± 0.57	90.83 ± 0.24	**93.42** ± **0.04**
4	*Chimpanzee versus Pig*	84.77 ± 0.25	85.57 ± 0.25	85.37 ± 0.15	85.09 ± 0.54	**86.29** ± **0.42**
5	*Chimpanzee versus Hippopotamus*	87.35 ± 0.06	86.83 ± 0.26	87.11 ± 0.29	87.72 ± 0.48	**89.84** ± **0.38**
6	*Chimpanzee versus Humpback whale*	98.25 ± 0.06	97.61 ± 0.10	**98.64** ± **0.39**	96.73 ± 0.31	98.19 ± 0.18
7	*Chimpanzee versus Raccoon*	83.60 ± 0.13	83.62 ± 0.16	86.07 ± 0.21	83.39 ± 0.49	**87.33** ± **0.42**
8	*Chimpanzee versus Rat*	85.52 ± 0.24	85.74 ± 0.54	86.50 ± 0.07	85.33 ± 0.26	**88.15** ± **0.42**
9	*Chimpanzee versus Seal*	90.78 ± 0.39	90.10 ± 0.30	89.96 ± 0.07	**90.82** ± **0.13**	88.62 ± 0.31
10	*Giant panda versus Leopard*	95.49 ± 0.49	**95.69** ± **0.14**	93.83 ± 0.13	91.79 ± 0.03	95.40 ± 0.41
11	*Giant panda versus Persian cat*	90.85 ± 0.00	**93.64** ± **0.30**	93.16 ± 0.46	90.63 ± 0.38	93.47 ± 0.29
12	*Giant panda versus Pig*	86.07 ± 0.52	86.09 ± 0.35	86.44 ± 0.54	**87.75** ± **0.15**	85.19 ± 0.58
13	*Giant panda versus Hippopotamus*	91.31 ± 0.29	90.85 ± 0.40	91.88 ± 0.50	92.24 ± 0.29	**92.83** ± **0.14**
14	*Giant panda versus Humpback whale*	**99.45** ± **0.49**	98.34 ± 0.39	98.74 ± 0.26	97.48 ± 0.30	98.20 ± 0.53
15	*Giant panda versus Raccoon*	88.18 ± 0.30	90.36 ± 0.55	**90.43** ± **0.09**	88.74 ± 0.16	89.54 ± 0.03
16	*Giant panda versus Rat*	84.63 ± 0.51	**88.18** ± **0.51**	87.41 ± 0.17	85.62 ± 0.30	86.93 ± 0.06
17	*Giant panda versus Seal*	90.07 ± 0.49	**91.30** ± **0.22**	90.86 ± 0.26	89.66 ± 0.30	90.09 ± 0.14
18	*Leopard versus Persian cat*	**95.09** ± **0.58**	93.38 ± 0.55	94.40 ± 0.06	94.82 ± 0.00	93.85 ± 0.07
19	*Leopard versus Pig*	**88.62** ± **0.12**	87.31 ± 0.10	88.10 ± 0.18	85.20 ± 0.31	86.81 ± 0.09
20	*Leopard versus Hippopotamus*	91.04 ± 0.04	91.51 ± 0.20	92.39 ± 0.53	90.74 ± 0.18	**94.06** ± **0.26**
21	*Leopard versus Humpback whale*	97.02 ± 0.00	98.21 ± 0.14	**99.04** ± **0.49**	98.19 ± 0.47	98.94 ± 0.05
22	*Leopard versus Raccoon*	**82.90** ± **0.11**	80.42 ± 0.29	81.46 ± 0.19	82.31 ± 0.13	82.65 ± 0.54
23	*Leopard versus Rat*	85.85 ± 0.52	87.91 ± 0.51	86.87 ± 0.19	87.65 ± 0.34	**88.85** ± **0.08**
24	*Leopard versus Seal*	93.02 ± 0.46	92.21 ± 0.12	94.13 ± 0.48	95.08 ± 0.48	**95.13** ± **0.04**
25	*Persian cat versus Pig*	**82.51** ± **0.27**	79.40 ± 0.05	80.76 ± 0.14	81.12 ± 0.01	81.94 ± 0.14
26	*Persian cat versus Hippopotamus*	92.02 ± 0.36	**94.00** ± **0.13**	92.83 ± 0.37	93.67 ± 0.26	93.56 ± 0.51
27	*Persian cat versus Humpback whale*	94.90 ± 0.23	98.37 ± 0.23	97.06 ± 0.08	**99.01** ± **1.00**	96.18 ± 0.18
28	*Persian cat versus Raccoon*	90.33 ± 0.23	**92.18** ± **0.39**	90.76 ± 0.51	90.70 ± 0.17	91.71 ± 0.39
29	*Persian cat versus Rat*	62.81 ± 0.21	63.04 ± 0.04	65.04 ± 0.37	64.07 ± 0.04	**67.80** ± **0.11**
30	*Persian cat versus Seal*	87.09 ± 0.44	87.47 ± 0.18	86.34 ± 0.49	87.29 ± 0.30	**89.21** ± **0.36**
31	*Pig versus Hippopotamus*	75.30 ± 0.06	76.38 ± 0.50	**76.71** ± **0.19**	74.16 ± 0.49	76.56 ± 0.48
32	*Pig versus Humpback whale*	94.30 ± 0.18	94.42 ± 0.30	96.74 ± 0.00	**97.13** ± **0.36**	96.28 ± 0.16
33	*Pig versus Raccoon*	79.55 ± 0.09	78.02 ± 0.15	80.07 ± 0.16	79.55 ± 0.05	**81.39** ± **0.49**
34	*Pig versus Rat*	70.72 ± 0.33	70.25 ± 0.41	72.98 ± 0.33	74.23 ± 0.19	**74.68** ± **0.11**
35	*Pig versus Seal*	77.88 ± 0.16	78.72 ± 0.20	78.08 ± 0.32	78.56 ± 0.56	**79.66** ± **0.07**
36	*Hippopotamus versus Humpback whale*	92.54 ± 0.17	93.70 ± 0.14	92.62 ± 0.00	94.02 ± 0.54	**94.15** ± **0.28**
37	*Hippopotamus versus Raccoon*	88.30 ± 0.17	83.36 ± 0.09	86.36 ± 0.47	87.68 ± 0.46	**89.60** ± **0.50**
38	*Hippopotamus versus Rat*	84.01 ± 0.59	**85.90** ± **0.17**	83.95 ± 0.05	83.59 ± 0.12	82.86 ± 0.06
39	*Hippopotamus versus Seal*	71.13 ± 0.49	**72.74** ± **0.03**	71.73 ± 0.47	69.87 ± 0.46	70.76 ± 0.50
40	*Humpback whale versus Raccoon*	96.57 ± 0.45	**98.59** ± **0.43**	97.08 ± 0.40	96.96 ± 0.28	96.29 ± 0.28
41	*Humpback whale versus Rat*	**96.54** ± **0.38**	96.01 ± 0.34	95.32 ± 0.29	95.25 ± 0.10	94.65 ± 0.04
42	*Humpback whale versus Seal*	84.21 ± 0.41	86.30 ± 0.38	85.97 ± 0.52	83.39 ± 0.31	**87.58** ± **0.10**
43	*Raccoon versus Rat*	78.09 ± 0.04	78.27 ± 0.28	77.96 ± 0.43	78.61 ± 0.02	**79.33** ± **0.38**
44	*Raccoon versus Seal*	92.05 ± 0.42	**92.51** ± **0.34**	90.60 ± 0.20	91.44 ± 0.52	89.36 ± 0.08
45	*Rat versus Seal*	77.29 ± 0.19	79.03 ± 0.38	77.39 ± 0.19	77.29 ± 0.02	**80.54** ± **0.44**
	Average	87.32	87.75	87.81	87.45	**88.38**

Table 5. **AwA dataset.** AP results for one-vs-one classification. Best average precisions are highlighted in boldface. The values in this table have been obtained by conducting the same experiment performed to obtain Table 4, with the difference that a Gaussian kernel was used in place of the linear kernel.