

# PARTICLE SWARM OPTIMIZATION (PSO) OF POWER ALLOCATION IN COGNITIVE RADIO SYSTEMS WITH INTERFERENCE CONSTRAINTS

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## Abstract

Cognitive radio is used for enhancement of spectrum efficiency. Although many works have been accomplished on the power allocation of cognitive radio, limited efforts have considered evolutionary algorithms. In this paper, we study this problem in the cognitive radio networks where interference constraints are defined for protection of quality of service (QoS) for both primary and secondary users. Utilities defined as functions of the signal-to-interference-plus-noise ratio (SINR) are matched for each secondary user which meets Nash's axioms. In general, the region of utilities that meets the constraints is non-convex. It is possible to make simplifications, generate a convex region, and then use common convex optimization approaches to obtain a solution. However, Particle Swarm Optimization (PSO) does not need such simplifications and thus its results are superior to those of the convex optimization methods. PSO is an evolutionary algorithm based on social intelligence, utilized in many optimization problems. PSO is a global optimizations algorithm that does not require the objective function be differentiable as required in classic optimization methods.

**Keywords:** Cognitive radio, Power allocation, Particle Swarm Optimization (PSO).

## 1 Introduction

As usage of wireless communication is becoming significant, requirement for unused spectrum is enhanced. Fixed policy of assignment makes spectrum inefficient. In 2002, Federal Communications Commission's (FCC's) reports that only 15% to 85% of licensed spectrum is used. Therefore, we should be able to improve utilization of the spectrum. Cognitive radio (CR) is a solution for this inefficiency which is originally nominated by Mitola [1]. In the early model of this system, users try to sense the underutilized spectrum which

is named hole, and then transmit the information on these holes [2], [3]. Vast exciting works have been performed on CR networks architectures, spectrum-sensing and pricing tactics [4], [5].

One aspect of research in the cognitive radio is system allocation of resources, which is a major problem because overall efficiency is directly affected by the allocation [6]. Based on the definition of cognitive radio, static spectrum utilization is against dynamic spectrum sharing (DSS). DSS is a way to improve inefficient static spectrum utilization by permitting users to dynamically sense and use white spaces in spectrum. Some of these schemes for spectrum sharing are hierarchical-access methods. In these methods, there are primary users (PUs) that have permission to access spectrum freely and secondary users (SUs) that dynamically use spectrum whenever possible [7].

There are mainly two scenarios for modeling of interaction between: first, underlay and second overlay techniques.

In the spectrum overlay technique, there is a spectrum management principle where a primary user permits a secondary user to use a channel only when it is empty. Spectrum overlay techniques are based on detect and avoid mechanism. The secondary user senses the frequency spectrum, if a primary user is active on a channel, then secondary user will not use this channel.

In the underlay technique, we consider users that use the shared spectrum and satisfy the predescribe thresholds for interference. The thresholds protect both primary and secondary users from harmful decreases of Quality of Service (QoS) [8], [9].

In general, in the underlay technique, the problem of resource allocation with these thresholds is a non-convex optimization problem. Previous schemes for solving this problem are based on relaxing the constraints on the signal-to-

interference-plus-noise ratio (SINR). Therefore, these relaxations translate the region of optimization into a convex set. However, they do not provide the best solution.

On the other hand, there are some global optimization approaches like Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) that do not require the objective function be differentiable. Therefore, these approaches are suitable for the field of cognitive radio. In the area of cognitive radio networks, an evolutionary algorithm has been investigated in [10] for the sensing aspect but we consider these algorithms for solving the problem of power allocation.

The Particle Swarm Optimization (PSO) is an algorithm inspired by nature, utilized in many fields of optimization [11], [12]. PSO is a computational method that optimizes a problem iteratively. In this paper, we use PSO to optimize a SINR utility and achieve better results than the other approaches.

The rest of this paper is organized as follows. In Section II, we describe the system. Then, we formulate the problem based on the SINR-utility function. In Section III, we describe PSO. In Section IV, simulation results are presented and in Section V, we present discussion and future work.

## 2 System model

Consider a scenario that  $N$  secondary users challenge to use the spectrum in a primary network that contains  $M$  users. The primary users are licensed to use the spectrum freely. A secondary user uses a spread spectrum model for sending information in the underlay manner and all users experience Rayleigh fading. As shown in Fig. 1, there exists a channel gain between the receiver and transmitter of each user which causes multiple access interference.

In Fig. 1,  $h_{s,i}$ ,  $h_{i,k}^s$ ,  $h_{p,k}$  and  $h_{k,i}^p$  respectively represent channel gain between the  $i$ th transmitter and receiver of SUs, the  $i$ th transmitter of SUs and the  $k$ th transmitter of PUs, the  $k$ th transmitter and receiver of PU and the  $k$ th transmitter of PU and the  $i$ th receiver of SUs. Indeed,  $\{h_{j,i}^c\}_{i \neq j}$  are representations of interference between SUs.  $y_{s,i}$  and  $y_{p,k}$  are respectively the received signal at the receiver of the  $i$ th SUs and the  $k$ th PUs:

$$y_{s,i} = x_{s,i}h_{s,i} + \sum_{j=1, j \neq i}^N h_{j,i}^c x_{s,j} + \sum_{k=1}^M h_{k,i}^p x_{p,k} + n_{s,i}$$

$$y_{p,k} = x_{p,k}h_{p,k} + \sum_{i=1}^N h_{i,k}^s x_{s,i} + n_{p,k} \quad (1)$$

For protection of PUs, we consider constraints on the interference, caused by SUs which is similar to [9]. Formal structure of this limitation is given by

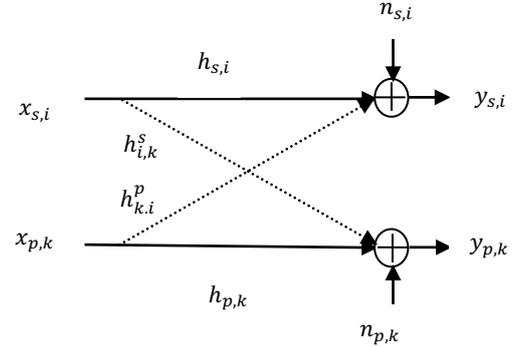


Figure 1 Fading channel between PUs and SUs.

$$\sum_{i=1}^N p_i f_{i,k}^s \leq Z_k, \quad k = 1, 2, \dots, M \quad (2)$$

Similar to [9], we also restrict the power of each user to guarantee minimum of QoS. In other words, we define  $\gamma_{i,min}$  for minimum of signal to interference and noise ratio (SINR) of a user.

The formulation of SINR is:

$$\gamma_i = \frac{p_i h_{s,i}}{\sum_{j=1, j \neq i}^N p_j h_{j,i}^c + \sum_{k=1}^M p_{p,k} h_{k,i}^p + \sigma_{s,i}^2} \quad (3)$$

where  $\sigma_{s,i}^2$  is the noise variance. Our purpose is to generate the following condition for every user:

$$\gamma_i \geq \gamma_{i,min} \quad \forall i \in 1, \dots, N \quad (4)$$

For a secondary user, we should define the utility function based on SINR, which we name SINR-utility function. This function has appropriate properties, because we consider restriction on SINR. In addition, it satisfies all the Nash axioms [13], [14].

$R = (\gamma_1, \gamma_2, \dots, \gamma_N)$  and  $R_{min} = (\gamma_{1,min}, \gamma_{2,min}, \dots, \gamma_{N,min})$  respectively represent SNRs of secondary users and minimum required SNRs of secondary users which  $R \in S$  and  $S$  is feasible set of  $R$ .

Definition: we will say that solution of problem  $F(R, R_{min})$  meets Nash axioms if the following conditions are satisfied [15].

- 1) *Individual Rationality*:  $R_i > R_{min,i}$
- 2) *Feasibility*:  $R \in S$
- 3) *Independence of Irrelevant Alternatives*: If  $S' \in S$  and  $R \subseteq S'$  then  $F = F'$  in which  $F'$  is a problem that considers  $S'$  instead of  $S$ .
- 4) *Independence of Linear Transformations*: If we apply linear transformation  $H$  to  $R, R_{min}$  then the problem is converted to problem that feasible set is  $H(S)$  and constraints are  $H(R_{min})$

5) *Symmetry*: It is invariant under all exchanges of agents.

As illustrated in [15], this formulation satisfies all of the Nash axioms.

$$u_i(p_1, p_2, \dots, p_N) = \log(\gamma_i - \gamma_{i,min}) \quad (5)$$

Therefore, the overall aim of cooperative formulation is to maximize:

$$\sum_{i=1}^N \log(\gamma_i - \gamma_{i,min}) \quad (6)$$

Subject to:

$$\begin{aligned} \gamma_i &\geq \gamma_{i,min} \\ \sum_{i=1}^N p_i f_{i,k}^s &\leq Z_k, k = 1, 2, \dots, M \\ \sum_{i=1}^N p_i &= p_{max} \end{aligned} \quad (7)$$

In general, the solution to (6) lies in a non-convex region because of limitation for interference of each primary user. Therefore, we cannot solve it using convex optimization methods. In [9], the authors present an appropriate pricing on the interference which means that if the power of a user increases, a higher cost should be paid [9]. By this approach, the limitation for interference of each primary user is removed from the formulation and we achieve a convex region. On the other hand, in our proposed algorithm, we solve the original problem and we compare our results to those of [9].

### 3 Particle swarm optimization and SINR utility

Particle swarm optimization (PSO) is inspired by social behavior of bird flocking or fish schooling, originally introduced by Eberhart and Kennedy [16] in 1995. PSO does not use the gradient of the objective function, therefore, PSO does not require for the objective function to be differentiable as is required by classic optimization methods. Therefore, PSO is suitable for optimization problems that are relatively irregular, noisy, or dynamic.

In PSO, there are some particles that search the space of solutions. In each step, particles change their positions based on the knowledge of themselves and other particles. Thus, positions of particles in swarm influence each particle. After several steps, particles move towards an optimum.

Each particle updates its position based on the following factors: its best solution (*Pbest*), a best solution of swarm (*gbest*), and a best solution of its neighbors (*nbest*).

*Pbest* is the best position that each particle found by the previous step. Each particle has a special *Pbest*. *gbest* is the best position that all of particles found by previous step. *gbest* is same for all of particles. We can consider some neighbors for each particle. *nbest* is the best position that all of neighbors of each particle found by previous step. Each particle has a special *nbest*.

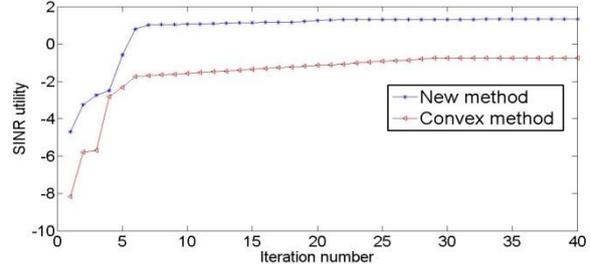


Figure 2 Comparison between the algorithm based on PSO and the algorithm presented in [9]. The algorithm base on PSO has better results due to the fact that it does not relax the constraints

There are several means to initialize positions of particles. One of them used in this paper is random initialization. In random initialization, particles are placed in random positions in the space. The update equation of positions is:

$$\begin{aligned} V &= c_0 X + c_1 r_1 \times (pbest - X) + \\ &\quad c_2 r_2 \times (gbest - X) + c_3 r_3 \\ &\quad \times (nbest - X) \\ X &= X + V \end{aligned} \quad (8)$$

Where  $c_0, c_1, c_2$  and  $c_3$  are constants and  $r_1, r_2$  and  $r_3$  are random numbers between 0 and 1. Also,  $V$  is velocity of each particle. In this study, we consider  $c_0 = 0, c_1 = 1.5, c_2 = 1.5$ , and  $c_3 = 0$  (which these amounts show the best results rather than other amounts). As mentioned before, the problem is to maximize SINR utility (Eq. (6)) under certain conditions. For doing this,  $p$  particles with random positions are produced. Then, their fitness (Eq. (6)) is calculated and *pbest* and *gbest* are obtained (we consider  $c_3 = 0$  in Eq. (8)) therefore, we do not calculate *nbest*). According to Eq. (8), velocity of particles is obtained and their positions are updated. This procedure is continued iteratively until a stopping condition is satisfied. Our stopping condition is that the number of iteration reaches a maximum or the increase of the fitness (Eq. (6)) is smaller than a given threshold ( $k$  denotes the iteration number):

$$|gbest^k - gbest^{k-1}| < \epsilon$$

In this study, the first one is chosen because the second one may be trapped in a local optimum.

Also, in each iteration, the constraints (Eq. (7)) are checked and the particles are forced to remain in the acceptable space.

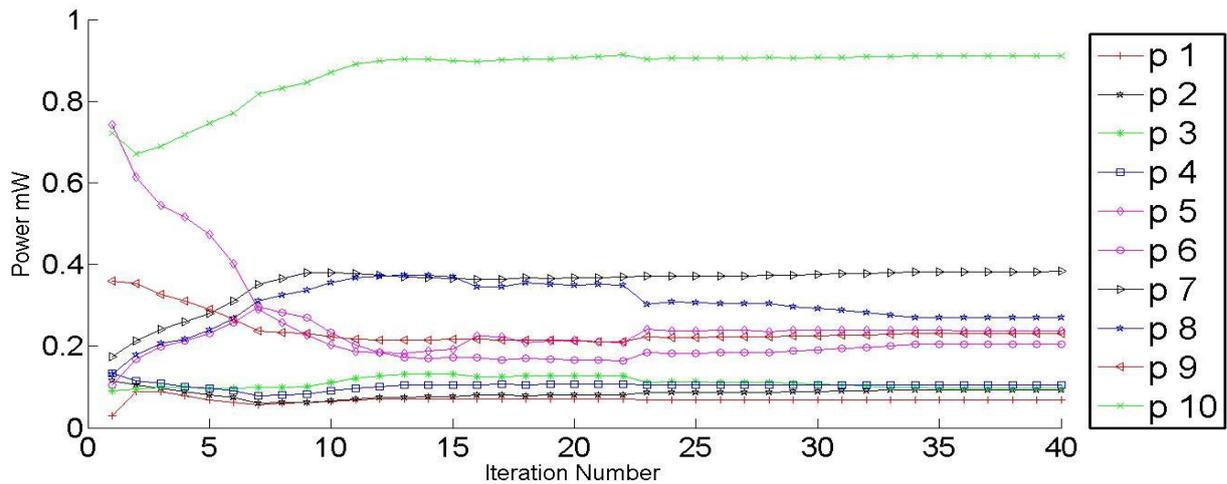


Figure 3 The power of each SU reaches their optimum in about 15 iterations.

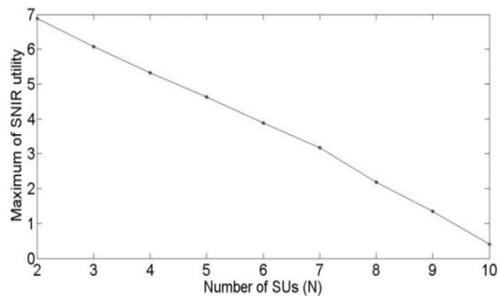


Figure 4 The optimum SINR utility for different number of SUs. The optimum SINR utility decreases when the number of SUs increases.

In this study, 50 particles are used with random position between 0 and 1. Since the SINR utility is very complex, we choose a relatively high number of particles.

#### 4 Results

This simulation is performed in MATLAB. The number of SUs and PU are chosen 10 and 1 respectively. The values of  $P_{max}$  and  $\gamma_{i,min}$  are 2.6 mW and 0.01, respectively. For all the communication and interference links, the channel gains are  $cd_{i,j}^{-\tau}$  for transmitter  $i$  and receiver  $j$ , where  $c = 0.097$  is constant and  $\tau = 4$  is the Rayleigh fading. The environment noise for both PU and SUs is assumed additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2 = 10^{-9} W$ . The bargain powers  $\alpha_i$  of all users are set to be equal. The comparison of results between the algorithm based on PSO and the approach presented in [9] is demonstrated in Fig. 2. In this figure, the amounts of SINR utility in each iteration are compared. Our results are better in all iterations due to the fact that the approach

presented in [9] relaxes the constraints but we do not.

The power of each SU is presented in Fig. 3. The power of each SU reaches its optimum gradually. It is obvious that the amounts of power of each SU are approximately reached their optimum in 15th iteration.

We change the number of SUs from 2 to 10 and calculate SINR utility for each case. The results are shown in Fig. 4. By increasing the number of secondary users, the total utility of system decreases because  $P_{max}$  is fixed and the assigned power of each user decreases. Moreover, interference increases as the number of secondary users increases.

The value of  $\gamma$  for each SU is presented in Table 1. Our results satisfy a constraint on  $\gamma$  (we consider  $\gamma_{min} = 0.01$ ).

As mentioned before, the results of approach based on PSO is better than the results of the approach presented in [9]. However, the speed of the approach based on PSO is lower. To solve this problem, we can use an improved approach based on PSO which is faster than the regular PSO [17].

We compare the basic approach based on PSO and an improved approach based on PSO [17].

Table 1 The value of  $\gamma$  of each SUs. Our results satisfy a constraint on  $\gamma$ .

|            |            |            |            |               |
|------------|------------|------------|------------|---------------|
| $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\gamma_5$    |
| 0.9938     | 0.9747     | 0.8598     | 0.9112     | 0.8129        |
| $\gamma_6$ | $\gamma_7$ | $\gamma_8$ | $\gamma_9$ | $\gamma_{10}$ |
| 0.8742     | 0.7731     | 0.7148     | 0.6804     | 6.0509        |

We consider 5 stages of elimination/addition and eliminate 8 particles and add 2 particles in each stage (described in [17]). To obtain reliable results, we perform these approaches 100 times under same conditions. The results are presented in Table 2. The result of the improved approach based on PSO is superior to that of the approach based on PSO. Also, the time of the improved PSO is less, around 58% compared to the time of the basic PSO.

Table 2 Average of maximum SINR utility. Improved PSO generates superior results than basic PSO.

| Improved PSO | PSO  |
|--------------|------|
| 1.41         | 1.12 |

## 5 Discussion and future work

In this paper, we have been solved the problem of optimal power allocation in cognitive radio systems. The proposed algorithm maximizes secondary networks' utilities that are based on SINR. The proposed approach is based on PSO which is an evolutionary algorithm. Similar to [9], our approach is stable but generates superior results. In our future work, we will eliminate the base station and assume that each user can make a decision using local information.

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