Free-form Textured Surfaces Registration by a Frequency Domain Technique

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Abstract

Free-form 3-D surfaces registration is a fundamental problem in 3-D imaging, tipically approached by extensions or variations of the ICP algorithm. This work presents a new frequency domain technique for 3-D view registration, totally different form an other recently proposed technique for 3-D motion estimation also, based on the Fourier transform. Conceively, the proposed method can give a non feature-based method for unsupervised registration of 3-D views. The obtained results are useful "per se" in applications targeted to visual quality or can serve as good starting point for the ICP algorithm when a higher precision is needed.

1 Introduction

Range data registration is tipically done by the ICP algorithm or its extensions [1, 2, 3]. Recently, it was proposed to approach this task by a frequency domain method [4, 5, 6]. This work proposes a new algorithmic idea which also operates in the frequency domain. The newly proposed method maintains the interesting characteristic related to the frequency domain of the previous one and it is numerically more robust. Since our technique is not based on features also this method can become a tool for unsupervised registration of range data.

2 Problem statement

Let $s_1(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^3$, be the range data of the surface of a 3-D object and let $s_2(\mathbf{x})$ be a rigidly translated, rotated and scaled version of $s_1(\mathbf{x})$, i.e.

$$s_2(\mathbf{x}) = s_1(R_\lambda^{-1}\mathbf{x} - \mathbf{t}) \,. \tag{1}$$

According to (1) $s_2(\mathbf{x})$ is first translated by the vector $\mathbf{t} \in \mathbb{R}^3$ then rotated by the matrix[1] $R \in SO(3)$, and finally scaled by the nonnegative real factor λ. Rotation by R and scaling by λ, are indicated by rotation with respect to

$$R_\lambda \doteq \lambda R \,. \tag{2}$$

[1] $SO(3) = \{R \in \mathbb{R}^{3\times 3}, R^{-1} = R^T, \det(R) = +1\}$ is the group of the 3×3 special orthogonal matrices.

Range data of this kind are associated to two free-form 3-D objects taken from two arbitrary points of view, or, in the view registration problem, they may be obtained from the common part of two partially overlapped views of an object taken at different distances. Scale factor λ is included for generality. Denote as

$$\mathcal{S}_i(\mathbf{k}) \doteq \mathcal{F}[s_i(\mathbf{x}) \mid \mathbf{k}] = \iiint_{-\infty}^{+\infty} s_i(\mathbf{x}) e^{-j2\pi\mathbf{k}^T\mathbf{x}}\, d\mathbf{x} \,, \tag{3}$$

where $\mathbf{k} = [k_x \;\; k_y \;\; k_z]^T$, the 3-D cartesian Fourier transform of $s_i(\mathbf{x})$, $i = 1, 2$; it is straightforward to prove that the two transforms are related as

$$\mathcal{S}_2(\mathbf{k}) = \lambda^3 \mathcal{S}_1(R_\lambda^T \mathbf{k}) e^{-j2\pi\mathbf{k}^T R_\lambda \mathbf{t}} \,. \tag{4}$$

Relationships (1) and (4) hold for 3-D surfaces as well as for 3-D solids. However, if the 3-D surfaces were modeled by impulsive functions supported on them, their Fourier transforms would have a lot of spurious high frequency content not suited to frequency domain methods. For this reason we preliminarily build 3-D solids from the range data defined by the 3-D surfaces in a way similar to that explained in [5, 6]. The solids companion to the range surfaces are defined as

$$\ell_i(\mathbf{x}) = \int_S s_i(\mathbf{x} - \mathbf{y}) h(\mathbf{y})\, d\mathbf{y} \,, \qquad i = 1, 2 \,, \tag{5}$$

with $h(\mathbf{y}) = e^{-c\|\mathbf{y}\|_2^2}$, S a small sphere centered around the origin and c suitable real nonnegative constant. With the data we used, which typically were matrices $128 \times 128 \times 128$, taking c in such a way that the sphere S with the 90% of the signal energy has 7 pixels diameter, it was found to be a good choice. Other choices for $h(\mathbf{y})$ and S would also be possible such as those proposed in [5]. The purpose of convolution (5) is to create small weighted solid regions around 3-D surface $s_i(\mathbf{x})$, with weight decreasing with the distance from $s_i(\mathbf{x})$. In this way solids $\ell_i(\mathbf{x})$ ideally maintain the same spatial information of the 3-D surfaces $s_i(\mathbf{x})$, without the mathematical difficulties associated to the impulsive supports.

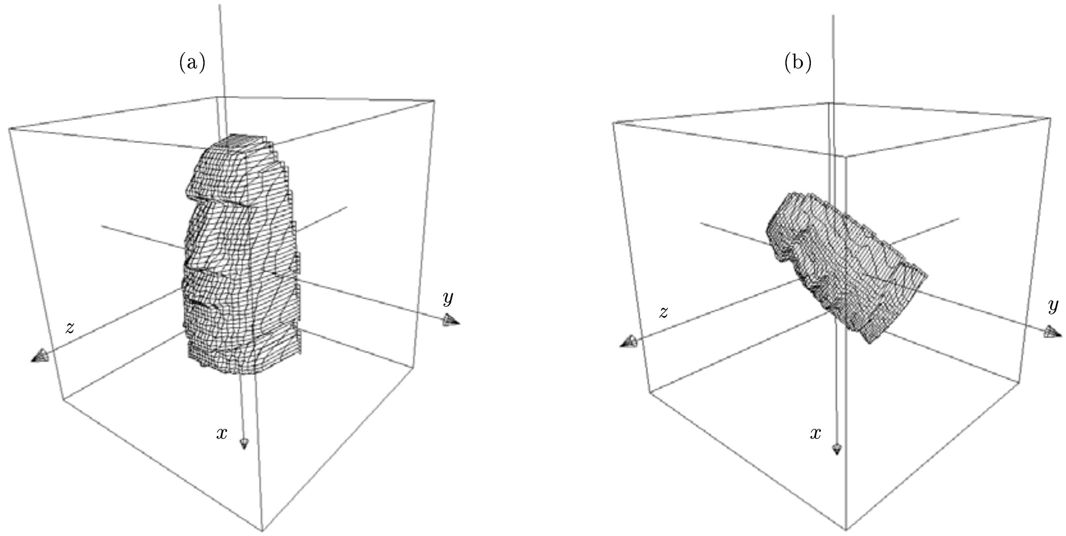

Figure 1: An example of range data surfaces related by a transformation of type (1): (a) original range data $s_1(\mathbf{x})$ (namely the file STAT1 of [9]); (b) the transformed version $s_2(\mathbf{x})$ obtained from $s_1(\mathbf{x})$ with $\mathbf{t} = [+2 \ -5 \ -10]^T$ (expressed in pixels), a rotation of $\varepsilon = 54.0°$ around a rotational axis positioned at $\varphi = 112.5°$ and $\vartheta = 26.5°$ (where φ and ϑ are refered to a coordinate system of the same type of (9)), and a scaling factor $\lambda = 0.72$.

The Fourier transforms of $\ell_i(\mathbf{x})$, $i = 1,2$, denoted as $L_i(\mathbf{k}) \doteq \mathcal{F}[\ell_i(\mathbf{x}) \mid \mathbf{k}]$, $i = 1,2$, from (3), satisfy (4) like $\mathcal{S}_i(\mathbf{k})$, $i = 1,2$. With reference to $L_i(\mathbf{k})$, $i = 1,2$, from (4) it can be noticed that the translational vector $\mathbf{t}$ affects only phases and not magnitudes. Therefore the magnitudes are related as

$$|L_2(\mathbf{k})| = \lambda^3 |L_1(R_\lambda^T \mathbf{k})| \,. \tag{6}$$

Relationship (6) can be used in order to determine the scaling factor λ and R. In fact in the frequency domain the estimation of λ, R and $\mathbf{t}$ can be decoupled and one can estimate first λ and R from (6) and then $\mathbf{t}$ from (4).

2.1 General structural properties of the Fourier transform of rotated signals

The scaling factor λ can be directly determined from (6) as

$$\lambda = \left(\frac{L_2(0)}{L_1(0)} \right)^{\frac{1}{3}} \,. \tag{7}$$

By defining $\mathcal{L}_1(\mathbf{k}) \doteq |L_1(\mathbf{k})|$ and $\mathcal{L}_2(\mathbf{k}) \doteq |L_2(\mathbf{k})|/\lambda^3$, equation (6) can be rewritten in normalized form as

$$\mathcal{L}_2(\mathbf{k}) = \mathcal{L}_1(R_\lambda^T \mathbf{k}) \,. \tag{8}$$

It is convenient to express (k_x, k_y, k_z) in spherical coordinates, namely

$$\begin{cases} k_x = \rho \cos\varphi \sin\vartheta \\ k_y = \rho \sin\varphi \sin\vartheta \\ k_z = \rho \cos\vartheta \end{cases} \tag{9}$$

with $0 \le \varphi < 2\pi$, $0 \le \vartheta \le \pi$, $\rho \ge 0$. Equation (8) in spherical coordinates can be written as

$$\begin{aligned} \bar{\mathcal{L}}_2(\rho,\varphi,\vartheta) &= \mathcal{L}_2(\rho\cos\varphi\sin\vartheta, \rho\sin\varphi\sin\vartheta, \rho\cos\vartheta) \\ &= \mathcal{L}_1(\rho(r_{11}\cos\varphi\sin\vartheta + r_{21}\sin\varphi\sin\vartheta \\ &\quad + r_{31}\cos\vartheta), \rho(r_{12}\cos\varphi\sin\vartheta \\ &\quad + r_{22}\sin\varphi\sin\vartheta + r_{32}\cos\vartheta), \\ &\quad \rho(r_{13}\cos\varphi\sin\vartheta + r_{23}\sin\varphi\sin\vartheta \\ &\quad + r_{33}\cos\vartheta)) \\ &= \bar{\mathcal{L}}_1(\tau(\varphi,\vartheta)\rho, \psi(\varphi,\vartheta), \chi(\varphi,\vartheta)) \\ &= \mathcal{L}_1(\tau(\varphi,\vartheta)\rho\cos\psi(\varphi,\vartheta)\sin\chi(\varphi,\vartheta), \\ &\quad \tau(\varphi,\vartheta)\rho\sin\psi(\varphi,\vartheta)\sin\chi(\varphi,\vartheta), \\ &\quad \tau(\varphi,\vartheta)\rho\cos\chi(\varphi,\vartheta)) \,, \end{aligned} \tag{10}$$

where r_{ij} denotes the (i,j)-th entry of R_λ. By comparing the arguments of the second expression of (10) with those of the last one, one obtains

$$\begin{cases} \tau(\varphi,\vartheta)\rho\cos\psi(\varphi,\vartheta)\sin\chi(\varphi,\vartheta) \\ \qquad = r_{11}\cos\varphi\sin\vartheta + r_{21}\sin\varphi\sin\vartheta + r_{31}\cos\vartheta \\ \tau(\varphi,\vartheta)\rho\sin\psi(\varphi,\vartheta)\sin\chi(\varphi,\vartheta) \\ \qquad = r_{12}\cos\varphi\sin\vartheta + r_{22}\sin\varphi\sin\vartheta + r_{32}\cos\vartheta \\ \tau(\varphi,\vartheta)\rho\cos\chi(\varphi,\vartheta) \\ \qquad = r_{13}\cos\varphi\sin\vartheta + r_{23}\sin\varphi\sin\vartheta + r_{33}\cos\vartheta \end{cases} \tag{11}$$

System (11) can be straightforwardly solved for $\tau(\varphi,\vartheta)$, $\psi(\varphi,\vartheta)$ and $\chi(\varphi,\vartheta)$ giving

$$\tau(\varphi,\vartheta) = \sqrt{\frac{\sum_{k=1}^{3}\left[(r_{1k}\cos\varphi + r_{2k}\sin\varphi)\tan\vartheta + r_{3k}\right]^2}{1+\tan^2\vartheta}} \tag{12}$$

$$\psi(\varphi,\vartheta) = \arctan\left(\frac{(r_{12}\cos\varphi + r_{22}\sin\varphi)\tan\vartheta + r_{32}}{(r_{11}\cos\varphi + r_{21}\sin\varphi)\tan\vartheta + r_{31}}\right) \tag{13}$$

$$\chi(\varphi,\vartheta) = \arccos\left(\frac{[(r_{13}\cos\varphi + r_{23}\sin\varphi)\tan\vartheta + r_{33}] \times \cos\vartheta}{\tau(\varphi,\vartheta)}\right) \tag{14}$$

By denoting with r'_{ij} the (i,j)-th entry of R, from (2) it is $r_{ij} = \lambda r'_{ij}$, the i-th row of R can be written as

$$\mathbf{r}_i = [r_{i1}\ r_{i2}\ r_{i3}] = \lambda[r_{i1}\ r_{i2}\ r_{i3}] = \lambda\mathbf{r}'_i\,. \tag{15}$$

Equation (12) can be rewritten as

$$\tau(\varphi,\vartheta) = \sqrt{\frac{\begin{array}{l}\|\mathbf{r}_3\|_2^2 + 2\cos\varphi\tan\vartheta\mathbf{r}_3\mathbf{r}_1^T + 2\sin\varphi\tan\vartheta \\ \times\,\mathbf{r}_2\mathbf{r}_3^T + 2\sin\varphi\cos\varphi\tan^2\vartheta\mathbf{r}_1\mathbf{r}_2^T \\ +\sin^2\varphi\tan^2\vartheta\|\mathbf{r}_2\|_2^2 + \cos^2\varphi\tan^2\vartheta\|\mathbf{r}_1\|_2^2\end{array}}{1+\tan^2\vartheta}} \tag{16}$$

since $\|\mathbf{r}_i\|_2^2 = \lambda^2$, $i=1,2,3$, and $\mathbf{r}_i\mathbf{r}_j^T = 0$ if $i \neq j$ and $i,j = 1,2,3$, expression (16) simply becomes

$$\tau(\varphi,\vartheta) = \lambda\,. \tag{17}$$

Result (17) allows one to simplify the denominator of expression (14).

3 Estimate of the rotation matrix R

Let us observe, that if one had three sets

$$(\psi^{(n)}, \chi^{(n)}, \varphi^{(n)}, \vartheta^{(n)}) \qquad n = 1,2,3\,, \tag{18}$$

satisfying equations (12), (13) and (14), then by defining

$$\beta_k^{(n)} \doteq r_{1k}\cos\varphi^{(n)}\sin\vartheta^{(n)} + r_{2k}\sin\varphi^{(n)}\sin\vartheta^{(n)} + r_{3k}\cos\vartheta^{(n)}\,, \tag{19}$$

where $k,n = 1,2,3$, expressions (12), (13) and (14) evaluated on (18) can be rewritten as

$$\begin{aligned}\lambda &= \sqrt{[\beta_1^{(n)}]^2 + [\beta_2^{(n)}]^2 + [\beta_3^{(n)}]^2}\,,\\ \psi^{(n)} &= \arctan\frac{\beta_2^{(n)}}{\beta_1^{(n)}}\,,\\ \chi^{(n)} &= \arccos\frac{\beta_3^{(n)}}{\lambda}\,.\end{aligned} \tag{20}$$

The expressions of $\beta_k^{(n)}$, $k,n = 1,2,3$, from (20) become

$$\begin{aligned}\beta_1^{(n)} &= \pm\lambda\sqrt{\frac{1-\cos^2\chi^{(n)}}{1+\tan^2\psi^{(n)}}}\,,\\ \beta_2^{(n)} &= \pm\lambda\tan\psi^{(n)}\sqrt{\frac{1-\cos^2\chi^{(n)}}{1+\tan^2\psi^{(n)}}}\,,\\ \beta_3^{(n)} &= \lambda\cos\chi^{(n)}\,.\end{aligned} \tag{21}$$

From positions

$$\begin{aligned}c_1^{(n)} &\doteq \cos\varphi^{(n)}\sin\vartheta^{(n)}\,,\\ c_2^{(n)} &\doteq \sin\varphi^{(n)}\sin\vartheta^{(n)}\,,\\ c_3^{(n)} &\doteq \cos\vartheta^{(n)}\,,\end{aligned} \tag{22}$$

system (19) can be rewritten as

$$\begin{bmatrix} c_1^{(1)} & 0 & 0 & c_2^{(1)} & 0 & 0 & c_3^{(1)} & 0 & 0\\ 0 & c_1^{(1)} & 0 & 0 & c_2^{(1)} & 0 & 0 & c_3^{(1)} & 0\\ 0 & 0 & c_1^{(1)} & 0 & 0 & c_2^{(1)} & 0 & 0 & c_3^{(1)}\\ c_1^{(2)} & 0 & 0 & c_2^{(2)} & 0 & 0 & c_3^{(2)} & 0 & 0\\ 0 & c_1^{(2)} & 0 & 0 & c_2^{(2)} & 0 & 0 & c_3^{(2)} & 0\\ 0 & 0 & c_1^{(2)} & 0 & 0 & c_2^{(2)} & 0 & 0 & c_3^{(2)}\\ c_1^{(3)} & 0 & 0 & c_2^{(3)} & 0 & 0 & c_3^{(3)} & 0 & 0\\ 0 & c_1^{(3)} & 0 & 0 & c_2^{(3)} & 0 & 0 & c_3^{(3)} & 0\\ 0 & 0 & c_1^{(3)} & 0 & 0 & c_2^{(3)} & 0 & 0 & c_3^{(3)} \end{bmatrix} \times \begin{bmatrix} r_{11}\\ r_{12}\\ r_{13}\\ r_{21}\\ r_{22}\\ r_{23}\\ r_{31}\\ r_{32}\\ r_{33} \end{bmatrix} = \begin{bmatrix} \beta_1^{(1)}\\ \beta_2^{(1)}\\ \beta_3^{(1)}\\ \beta_1^{(2)}\\ \beta_2^{(2)}\\ \beta_3^{(2)}\\ \beta_1^{(3)}\\ \beta_2^{(3)}\\ \beta_3^{(3)} \end{bmatrix}. \tag{23}$$

Expression (23) is linear with respect to the entries of R_λ. Note that because of the sign uncertainties on each $\beta_k^{(n)}$, $k=1,2,3$, of (21), expression (23) is associated to 8 linear systems. However it can be easily proved that four solutions of (23) produce R_λ with a negative determinant therefore they can be easily rejected by this check. The remaining four are pairwise identical, therefore the final disambiguation involves only two matrixes. The remaining two possible solutions for R_λ can be disambiguated by a phase correlation method presented next. Once one has R_λ then R can be simply obtained from (2).

The previous observation, resting upon the analysis of Section 2, suggests a procedure for estimating R, based on the determination of three sets of type (18). Such sets can be determined from the projections of $\bar{\mathcal{L}}_i(\rho,\varphi,\vartheta)$, $i=1,2$, with respect to ρ.

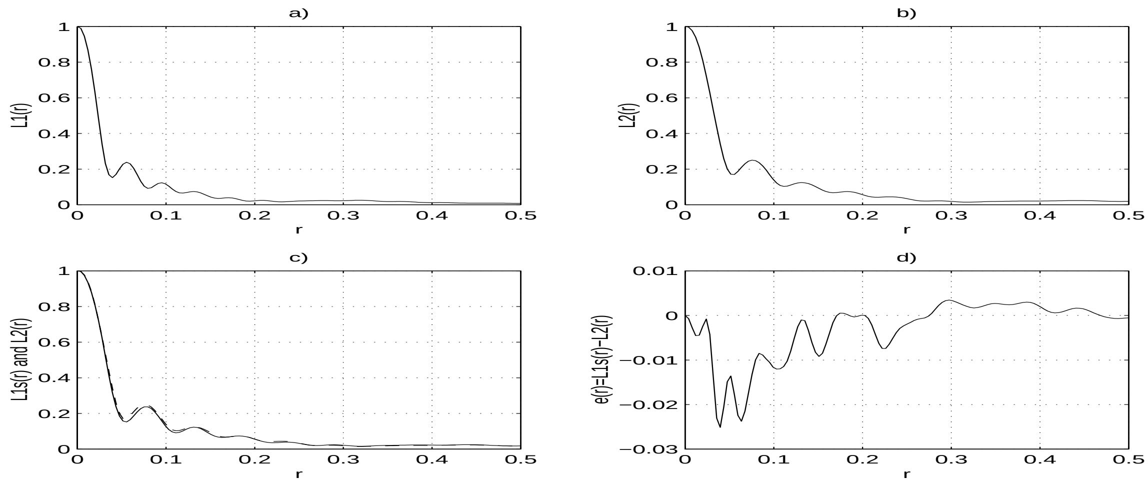

Figure 2: a) Function $\bar{\mathcal{L}}_1(\rho, \hat{\varphi}, \hat{\vartheta})$ with $\hat{\varphi} = 99.0°$ and $\hat{\vartheta} = 46.0°$; b) function $\bar{\mathcal{L}}_2(\rho, \hat{\psi}, \hat{\chi})$ with $\hat{\psi} = 126.0°$ and $\hat{\chi} = 46.5°$; c) superposition of stretched function $\bar{\mathcal{L}}_{1s}(\rho, \hat{\varphi}, \hat{\vartheta})$ and $\bar{\mathcal{L}}_2(\rho, \hat{\psi}, \hat{\chi})$ (dashed); d) error $e(\rho) = \bar{\mathcal{L}}_{1s}(\rho, \hat{\varphi}, \hat{\vartheta}) - \bar{\mathcal{L}}_2(\rho, \hat{\psi}, \hat{\chi})$.

Call

$$\mathcal{P}_i(\varphi, \vartheta) \doteq \int_0^\infty \bar{\mathcal{L}}_i(\rho, \varphi, \vartheta)\, d\rho\,, \qquad i = 1, 2\,, \tag{24}$$

and note that

$$\begin{aligned}\mathcal{P}_2(\varphi, \vartheta) &= \int_0^\infty \bar{\mathcal{L}}_1(\lambda\rho, \psi(\varphi, \vartheta), \chi(\varphi, \vartheta))\, d\rho \\ &= \frac{1}{\lambda}\mathcal{P}_1(\psi(\varphi, \vartheta), \chi(\varphi, \vartheta))\;.\end{aligned} \tag{25}$$

Since, from (25), projection $\mathcal{P}_2$ differs from $\mathcal{P}_1$ by the constant scale factor λ and by an angular modulation on ψ and χ, it seems plausible that the points of maximum of $\mathcal{P}_1$ within a small rotation, correspond also to the points of maximum of $\mathcal{P}_2$. Based on this concept one may use the following algorithm, inspired by [7] in order to define three sets of type (18).

Algorithm

1) find the M, $M \geq 3$, dyads of angles $(\varphi^{(m)}, \vartheta^{(m)})$, $m = 1, \ldots, M$, where $\mathcal{P}_1(\varphi, \vartheta)$ exhibits the M highest local maxima;

2) find the N, $N \geq 3$, dyads of angles $(\psi^{(n)}, \chi^{(n)})$, $n = 1, \ldots, N$, where $\mathcal{P}_2(\psi, \chi)$ exhibits the N highest local maxima;

3) let $E \doteq (e_{ij}) = 0$, $E \in \mathbb{R}^{M\times N}$, be the error matrix;

4) **for** $i = 1$ to $i = M$
   for $j = 1$ to $j = N$
   if $\lambda > 1$
   then stretch $\bar{\mathcal{L}}_2(\rho, \psi^{(j)}, \chi^{(j)})$
   $\bar{\mathcal{L}}_{2s}(\rho, \psi^{(j)}, \chi^{(j)}) = \bar{\mathcal{L}}_2(\rho/\lambda, \psi^{(j)}, \chi^{(j)})$
   $e_j(\rho) = \bar{\mathcal{L}}_1(\rho, \varphi^{(i)}, \vartheta^{(i)}) - \bar{\mathcal{L}}_{2s}(\rho, \psi^{(j)}, \chi^{(j)})$
   else stretch $\bar{\mathcal{L}}_1(\rho, \varphi^{(i)}, \vartheta^{(i)})$
   $\bar{\mathcal{L}}_{1s}(\rho, \varphi^{(i)}, \vartheta^{(i)}) = \bar{\mathcal{L}}_1(\lambda\rho, \varphi^{(i)}, \vartheta^{(i)})$
   $e_j(\rho) = \bar{\mathcal{L}}_{1s}(\rho, \varphi^{(i)}, \vartheta^{(i)}) - \bar{\mathcal{L}}_2(\rho, \psi^{(j)}, \chi^{(j)})$
   end
   $e_{ij} \doteq \int_0^\infty |e_j(\rho)|\, d\rho$
   end
   end

5) find the three dyads of indexes $(i^{(h)}, j^{(h)})$, $h = 1, 2, 3$, with $i^{(l)} \neq i^{(m)}$ if $l \neq m$ and $j^{(l)} \neq j^{(m)}$ if $l \neq m$, where E exhibits the three lowest absolute minima;

6) let $(\psi^{(j^{(h)})}, \chi^{(j^{(h)})}, \varphi^{(i^{(h)})}, \vartheta^{(i^{(h)})})$, $h = 1, 2, 3$, be the three sets with which one can solve system (23) with respect to r_{ij}.

4 Disambiguation of R and estimate of t

From the algorithm of the previous section we obtain two estimates for R, denotes as $\hat{R}_1$ and $\hat{R}_2$. Assume $\hat{R}_1 \in SO(3)$ to be equal R and $\hat{R}_2 \in SO(3)$ to be a matrix different than R. Compute

$$d'(\mathbf{x}) \doteq \ell_2(\lambda \hat{R}_1 \mathbf{x}) = \ell_1(\mathbf{x} - \mathbf{t})\,, \tag{26}$$

and

$$\begin{aligned} d''(\mathbf{x}) &\doteq \ell_2(\lambda \hat{R}_2 \mathbf{x}) = \ell_1(\lambda R_\lambda^{-1} \hat{R}_2 \mathbf{x} - \mathbf{t}) \\ &= \ell_1(R^{-1} \hat{R}_2 \mathbf{x} - \mathbf{t}) = d'(R'\mathbf{x}) \,, \end{aligned} \tag{27}$$

where $R' \doteq R^{-1}\hat{R}_2 \in SO(3)$ is a rotation matrix. Signal $d'(\mathbf{x})$ is a version of $\ell_1(\mathbf{x})$ translated by $\mathbf{t}$. Signal $d''(\mathbf{x})$ is a version $d'(\mathbf{x})$ rotated by R'. This fact suggests the use of phase correlation algorithm [8] not only for estimating shift $\mathbf{t}$ but also for solving disambiguation. Compute

$$\Phi'(\mathbf{k}) \doteq \frac{L_1^*(\mathbf{k}) D'(\mathbf{k})}{|L_1(\mathbf{k}) D'(\mathbf{k})|} = e^{-j2\pi \mathbf{k}^T \mathbf{t}} \,, \tag{28}$$

and

$$\begin{aligned} \Phi''(\mathbf{k}) &\doteq \frac{L_1^*(\mathbf{k}) D''(\mathbf{k})}{|L_1(\mathbf{k}) D''(\mathbf{k})|} \\ &= e^{j\{\arg[L_1^*(\mathbf{k})] + \arg[L_1(R'\mathbf{k})]\}} e^{-j2\pi \mathbf{k}^T R'^{-1} \mathbf{t}} \,, \end{aligned} \tag{29}$$

where $D'(\mathbf{k}) \doteq \mathcal{F}[d'(\mathbf{x})|\mathbf{k}]$ and $D''(\mathbf{k}) \doteq \mathcal{F}[d''(\mathbf{x})|\mathbf{k}]$. The inverse Fourier transform of $\Phi'(\mathbf{k})$ is

$$\phi'(\mathbf{k}) \doteq \mathcal{F}^{-1}[\Phi'(\mathbf{k})|\mathbf{x}] = \delta_{\mathbb{R}^3}(\mathbf{x} - \mathbf{t}) \,, \tag{30}$$

i.e. an impulsive function located at $\mathbf{t}$. The inverse Fourier transform of $\Phi''(\mathbf{k})$, denoted as $\phi'(\mathbf{x})$, is not an impulsive one. Therefore comparison between the peaks of $\phi'(\mathbf{x})$ and $\phi''(\mathbf{x})$ allows one to rule out solution $\hat{R}_2$ related to $\phi''(\mathbf{x})$, and vector $\mathbf{t}$ can be estimated from the position of the peak of $\phi'(\mathbf{x})$. For the test whose Figures refer to, the true parameters were, for the rotation matrix angles $\varphi = 112.5°$, $\vartheta = 26.5°$, $\varepsilon = 54.0°$, and for the translation vector $\mathbf{t} = [+2 \ -5 \ -10]^T$. The parameters estimated by the proposed procedure were $\hat{\varphi} = 113.5°$, $\hat{\vartheta} = 26.5°$, $\hat{\varepsilon} = 53.2°$and $\hat{\mathbf{t}} = [+2 \ -5 \ -10]^T$.

5 Conclusion

This work presents a new algorithm based on the frequency domain for estimating 3-D rotations and translations under scale changes. This method, although in the frequency domain, rests upon algorithmic ideas totally different than those of [5, 6] and it allows one to estimate also for scale factor changes, which the methods [5, 6] do not include. At the moment of this writing not enough experimental data are available for assessing the practical performance of the proposed method. Conceivably it can be applied to 3-D views registration in tasks requiring a limited precision or it can be used in order to obtain effective starting points for standard methods, which, as well-known, can give accurate solutions, once they are properly initiated [3].

Future work will aim to assess the robustness and the efficiency of the proposed 3-D registration method and to improve it by incorporating the provisions suggested by practical experimentation. An important aspect of the method, which is worth investigating, is the improvement on precision and robustness which one may gain by using texture information (easily obtainable by using also a digital photo-camera in the acquisition process) along with range data.

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